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MIE SCATTERING: A COMPUTER PROGRAM AND AN ATLAS

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16. ABSTRACT

A brief background of Mie Scattering is given, with applications in distributions of particles and effects of different lens apertures of the flux-collecting device. A computer program, MIESCA, which calculates amplitude functions, efficiency factors, intensities, and polarizations, is listed, with adaptations for tape filing in the Univac 1108 and for plotting intensities and polarizations versus scattering angle on the SC 4020 plotter. An Appendix on several numerical integrations and an Atlas of scattering intensity graphs are included.

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MIE SCATTERING: A COMPUTER PROGRAM AND AN ATLAS

SUMMARY

A computer program for calculating the Mie Scattering parameters is presented with applications in distributions of particle size and effects of different lens apertures of a detector. The computer program,* MIESCA, calculates the amplitude functions, efficiency factors, intensities, and polarizations. The program was adapted for magnetic tape filing in the Univac 1108 and for plotting intensities and polarization versus scattering angle on the SC 4020 plotter. An Appendix and an Atlas of scattering intensity graphs are included. This report provides a basis for a program to evaluate characteristics of the radiation scattered by a dilute cloud of particles.

INTRODUCTION

Electromagnetic radiation passing through a dilute cloud of particles is scattered and absorbed by the particles in the cloud. The characteristics of the scattered radiation are determined by the wavelength, λ, of the incident radiation, the complex refractive index, m, of the medium, and the size and shape of the discrete particles in the medium. Therefore, information about the state of the medium can be provided by the measurement and proper interpretation of the characteristics of the scattered radiation. The numerical determination of the characteristics of the scattered radiation for given particle models is of prime importance. This report provides the first step in a program for the evaluation of characteristics of the radiation scattered by a dilute cloud of particles. Programs for scattering by a single spherical particle (Mie Scattering) are presented.

T-027 (ATM Contamination Measurement) and S-073 (Gegenshein/ Zodiacal Light) are two photometric experiments assigned to the first AAP flights. The T-027 objective is to study the spacecraft contaminant cloud by observing the scattered light of the particles. The S-073 objective is to study the interplanetary dust by scattered light. The scattering programs presented and to be developed will be available for use in data analysis for the T-027 and S-073 flight experiments.

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* The basic program was developed by Brown Engineering Company under NASA Contract NAS8-20166.

MIE SCATTERING

Theory

In the T-027 and S-073 AAP experiments, light scattering by particles will prove to be important in data analysis. Therefore, this initial report provides an Atlas of Mie Scattering diagrams for different particle sizes to aid in the identification of possible contamination for T-027 and in synthesizing a zodiacal light model for S-073. Light scattering by spheres of arbitrary size and refractive index are described by Mie Scattering. The angular distribution of intensity of scattered radiation was obtained by Mie, and independently by others, who successfully applied Maxwell's equations with the appropriate boundary conditions. The basic expressions for the radiation scattered by a sphere of radius a, and of a material with a complex refractive index m, are [1]:

$$i_1 = \left| \sum_{n=1}^{\infty} \left\{ A_n \frac{d\,P_n(x)}{dx} + B_n \left[x \frac{d\,P_n(x)}{dx} - (1-x)^2 \frac{d^2 P_n(x)}{dx^2} \right] \right\} \right|^2 ,$$

$$i_2 = \left| \sum_{n=1}^{\infty} \left\{ A_n \left[x \frac{d\,P_n(x)}{dx} - (1-x^2) \frac{d^2 P_n(x)}{dx^2} \right] + B_n \frac{d\,P_n(x)}{dx} \right\} \right|^2 ,$$

$$K = \frac{2}{\alpha^2} \sum_{n=1}^{\infty} \frac{n^2 (n+1)^2}{2n+1} \left(\left| A_n \right|^2 + \left| B_n \right|^2 \right)$$

where $P_n(x)$ is a Legendre polynomial of order n, and $x = -\cos(\theta)$. A_n and B_n are amplitude functions (linear combinations of Ricatti-Bessel, Ricatti-Hankel functions and their derivatives, with arguments $\alpha = (2\pi a)/\lambda$ and $m\alpha$). The physical significance of i_1 and i_2, the intensity functions, is that they are the intensities of the two incoherent plane-polarized components of light scattered by a single particle (Fig. 1). When this particle is illuminated by randomly polarized light of unit intensity, then $(\lambda^2 i_1)/(8\pi^2)$ and $(\lambda^2 i_2)/(8\pi^2)$ are the radiant intensities of the scattered components with electric vectors perpendicular and parallel, respectively, to the plane of observation. The differential scattering cross section per particle is

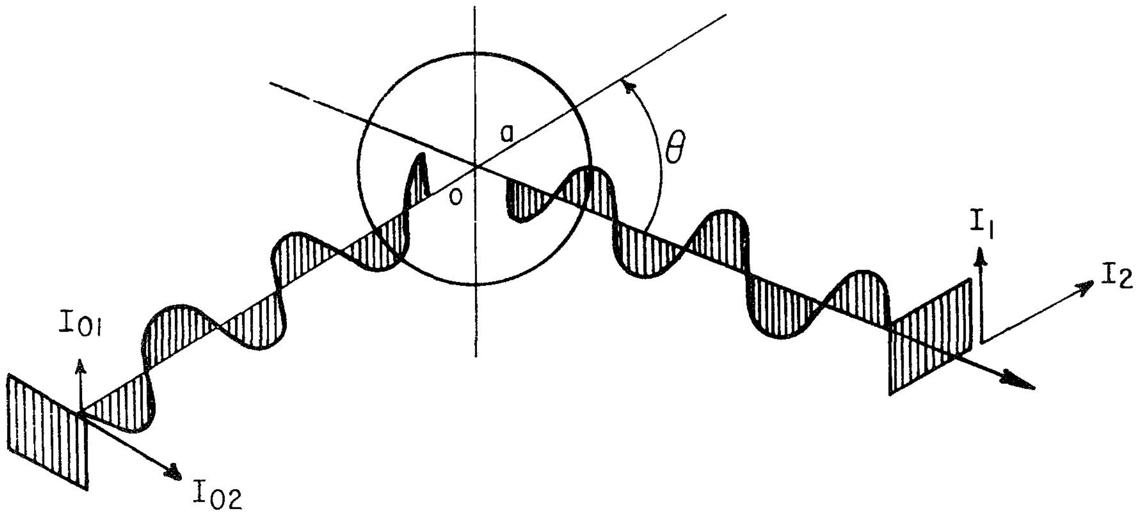


Figure 1. Physical representation of the intensities.

$$\frac{d\sigma}{d\Omega}(\theta,\lambda) = \frac{\lambda^2}{8\pi^2}(i_1 + i_2)$$

Q_{sca} is the scattering coefficient or efficiency, defined by the ratio of the scattering cross section to the geometric cross section of the particles. (The scattered wave intensity is obtained by integration of the radial component of the Poynting vector of the scattered wave over the surface of a sphere surrounding the particle.) The angular distribution function, $f(\theta)$, the fraction of scattered radiation directed into a solid angle in the direction of θ, is defined as

$$f(\theta) = \frac{i_1 + i_2}{2\pi\,\alpha^2\,Q_{sca}}$$

Substituting the expression for $f(\theta)$ into the expression for the differential scattering cross section yields:

$$\frac{d\sigma}{d\Omega}(\theta,\lambda) = \frac{\lambda^2\alpha^2}{4\pi}\,Q_{sca}\,f(\theta) \quad .$$

The Computer Program

The form of the general formulas is not amenable to computer computation, so numerical methods must be employed [1]. The most fundamental factors in the Mie solution are the amplitude functions, which can be calculated by the following formulas:

$$A_n = i^{2n+1} \frac{2n+1}{n(n+1)} \frac{\psi_n(\alpha) \dfrac{d\,\psi_n(\beta)}{d\beta} - m\ \psi_n(\beta) \dfrac{d\,\psi_n(\alpha)}{d\alpha}}{\zeta_n(\alpha) \dfrac{d\,\psi_n(\beta)}{d\beta} - m\ \psi_n(\beta) \dfrac{d\,\zeta_n(\alpha)}{d\alpha}}$$

$$B_n = -\,i^{2n+1} \frac{2n+1}{n(n+1)} \frac{\psi_n(\beta) \dfrac{d\,\psi_n(\alpha)}{d\alpha} - m\ \psi_n(\alpha) \dfrac{d\,\psi_n(\beta)}{d\beta}}{\psi_n(\beta) \dfrac{d\,\zeta_n(\alpha)}{d\alpha} - m\ \zeta_n(\alpha) \dfrac{d\,\psi_n(\beta)}{d\beta}}$$

where

ψ_n is a Ricatti-Bessel function, of order n, and

ζ_n is a Ricatti-Hankel function, of order n.

Also,

$$\alpha = \frac{2\pi a}{\lambda} \quad \text{and} \quad \beta = m\alpha = \alpha(m^* - ik) \quad ,$$

where the basic parameters are defined:

a = radius of a spherical particle

λ = wavelength of incident radiation

m = complex refractive index

m^* = real part of m

k = imaginary part of m (extinction coefficient of the particle material)

i = $(-1)^{\frac{1}{2}}$

From the amplitude functions, the efficiency factors for total extinction, scattering and absorption are computed:

$$Q_{ext} = \frac{2}{\alpha^2} \operatorname{Im} \sum_{n=1}^{\infty} n(n+1)(-1)^n (A_n - B_n)$$

$$Q_{sca} = \frac{2}{\alpha^2} \sum_{n=1}^{\infty} \frac{n^2(n+1)^2}{2n+1} (|A_n|^2 + |B_n|^2)$$

$$Q_{abs} = Q_{ext} - Q_{sca}$$

The computer input is composed of the scattering angle, the refractive index of the medium, and α. The program computes and prints out the efficiency factors, both intensities i_1 and i_2, their sum, and the polarization. (The polarization is defined as: $i_1 - i_2/i_1 + i_2$.) An example of the computer output is given in Table 1, for which α = ALT = 5.0, θ = XTH1 = 0 degree (5 degrees) 180 degrees, and m* = EM = 1.33. An SC 4020 plot for these values is also included in the Atlas. Appendix A gives the details for the graphical display using the Univac 1108 — SC 4020 plotter which was used to produce the Atlas (Appendix C). Appendix A gives the 1108 user information concerning the Mie computer programs.

APPLICATIONS

Dilute clouds of particles are normally polydispersed and are observed through finite apertures; therefore, integrations over θ and α are necessary for data analysis. This section considers integrations over particle size distribution and finite angular apertures.

TABLE 1. SAMPLE OF MIESCA PRINTOUT FOR α = 5.0, m = 1.33

DATA SET NO. 5 MIE SCATTERING

REFRACTIVE INDEX RE(M) = 1.330 IM(M) = .000
PARAMETER ALPHA = 5.00

AMPLITUDE FUNCTIONS

N	RE(AN)	IM(AN)	RE(BN)	IM(BN)
1	.119825	-1.490366	.246441	1.458355
2	.025744	.332537	.194088	-.785368
3	.153892	-.539430	-.075414	.573415
4	-.213585	.295756	.141358	-.400051
5	.118090	-.043398	-.086858	.021881
6	-.019586	.001244	.008516	-.000235
7	.002326	-.000020	-.000735	.000002
8	-.000221	.000000	.000053	-.000000
9	.000017	-.000000	-.000003	.000000
10	-.000001	.000000	.000000	-.000000
11	.000000	-.000000	-.000000	.000000

EFFICIENCY FACTOR FOR EXTINCTION QEXT = 3.59103

EFFICIENCY FACTOR FOR SCATTERING QSCAT = 3.59103

EFFICIENCY FACTOR FOR ABSORPTION QABS = .00000

TABLE 1. (Concluded)

SCAT ANGLE THETA(DEG)	INTENSITY FUNCTIONS I1	I2	DEGREE OF POLARIZATION P	I1+I2
0	585.897980	585.897980	.000000	1171.795959
5	557.296188	556.870422	.000382	1114.166611
10	478.544411	477.507244	.001085	956.051651
15	368.327080	367.919376	.000554	736.246452
20	250.602343	252.990770	-.004743	503.593113
25	147.040073	153.853794	-.022645	300.893867
30	71.223001	82.188796	-.071479	153.411797
35	26.509806	39.204371	-.193178	65.714177
40	7.696425	18.649259	-.415735	26.345685
45	5.070577	11.528659	-.389059	16.599236
50	8.814129	10.075082	-.066755	18.889211
55	12.069953	9.630737	.112403	21.700689
60	11.961829	8.508268	.168713	20.470097
65	8.852539	6.780700	.132528	15.633239
70	4.710728	5.034490	-.033223	9.745218
75	1.509934	3.660370	-.415921	5.170304
80	.255789	2.714107	-.827746	2.969896
85	.792422	2.075666	-.447421	2.868089
90	2.192232	1.626137	.148256	3.818369
95	3.378646	1.313296	.440191	4.691942
100	3.654802	1.123927	.529613	4.778729
105	2.941924	1.038014	.478377	3.979938
110	1.693264	1.015446	.250236	2.708711
115	.584198	1.015555	-.269640	1.599753
120	.150871	1.020906	-.742492	1.171777
125	.545231	1.041527	-.312773	1.586758
130	1.503082	1.098189	.155652	2.601271
135	2.516375	1.200819	.353911	3.717194
140	3.107129	1.338979	.397685	4.446108
145	3.059922	1.489183	.345285	4.549105
150	2.500450	1.630480	.210599	4.130930
155	1.792449	1.755150	.010514	3.547599
160	1.320296	1.867200	-.171578	3.187497
165	1.280683	1.971697	-.212464	3.252380
170	1.595284	2.064614	-.128236	3.659898
175	1.985158	2.131574	-.035566	4.116732
180	2.156285	2.156285	.000000	4.312570

Distributions

The zodiacal cloud is composed of many different sizes of particles, whose distribution is generally assumed to be of an exponential form:

$$N = Na_0 \left(\frac{a}{a_0}\right)^{-p} \approx N\alpha_0 \left(\frac{\alpha}{\alpha_0}\right)^{-p}$$

where a and α are as previously defined, and p is dependent on a_{min}, a_{max}, and some "breakpoint" a_B between them [2]. Other types of distributions, such as aerosol distributions, will be considered for T-027.

The first distribution examined was the exponentially decreasing function $e^{-\alpha}$. The choice of $e^{-\alpha}$ is convenient because,

$$\int_0^\infty e^{-\alpha} \, d\alpha = 1$$

So, the percentage of each size particle in the distribution can be found easily on the basis of its fractional part of the whole.

When calculating an intensity for particles with a size distribution, the different intensities for each α must be weighted according to the distribution. For a particular θ,

$$I_{total}(\theta) = \int_{\alpha_{min}}^{\alpha_{max}} I(\alpha, \theta) \, e^{-\alpha} \, d\alpha \ .$$

Then, this must be normalized to give

$$\int_{\alpha_{min}}^{\alpha_{max}} e^{-\alpha} \, d\alpha = 1$$

By Gauss-Legendre Quadrature, this can be numerically integrated in a Fortran program. (See Appendix B.) An example for $m = 1.3 - 0.0i$, $\alpha_{min} = 0.1$ and $\alpha_{max} = 10.0$ is in Table 2.

Apertures

The angular aperture of the objective lens of a photometric system distorts the observed scattering distribution. This effect is being studied in connection with the T-027 data analysis. It is impossible to record the intensity at a single θ, just as it is impossible to measure the integral or "area" under a point. An integral must be taken over some interval. The interval in this study is the lens aperture. The intensity is calculated over the whole aperture and the "integrated" intensity distribution is determined. Effects of changing the angular aperture of the flux collecting device must be studied to insure correct interpretation of measured scattering distributions. Widening the lens aperture smoothes the intensity distributions and reduces the number of maxima.

The flux, Φ, is the integral of the intensity over the solid angle. Gucker and Tuma [3] have developed a method of modifying the flux equation,

$$\Phi(\eta,\omega) = \left(\frac{\lambda}{2\pi}\right)^2 \int_{\theta=\eta-\omega}^{\theta=\eta+\omega} (i_1 + i_2)\sin\theta \cos^{-1}\left(\frac{\cos\omega - \cos\theta\cos\eta}{\sin\theta\,\sin\eta}\right) d\theta$$

for computation, where η = angle of observation, ω is the angle of aperture, and θ is the scattering angle (Fig. 2). After normalizing (by dividing through by $\left(\frac{\lambda}{2\pi}\right)^2$), the flux equation becomes:

$$\Phi(\eta,\omega) = \sum_{k=1}^{n} \left[i_1(\theta_k, \alpha, m) + i_2(\theta_k, \alpha, m) \right] \cdot a_k(\eta,\omega)$$

$$a_k(\eta,\omega) = \frac{A_k(\eta,\omega)}{2\pi R^2 (1 - \cos\omega)}$$

TABLE 2. NORMALIZED DISTRIBUTION FOR m = 1.3 - 0.0i

THETA	REFRACTIVE INDEX		GAUSSIAN INTEGRAL
.00	1.330	.000 I	500.45267
5.00	1.330	.000 I	456.38535
10.00	1.330	.000 I	349.38321
15.00	1.330	.000 I	232.01636
20.00	1.330	.000 I	142.33855
25.00	1.330	.000 I	86.94949
30.00	1.330	.000 I	54.83612
35.00	1.330	.000 I	35.36060
40.00	1.330	.000 I	23.25827
45.00	1.330	.000 I	15.83237
50.00	1.330	.000 I	11.13292
55.00	1.330	.000 I	7.98860
60.00	1.330	.000 I	5.86744
65.00	1.330	.000 I	4.44560
70.00	1.330	.000 I	3.45446
75.00	1.330	.000 I	2.72970
80.00	1.330	.000 I	2.20637
85.00	1.330	.000 I	1.83888
90.00	1.330	.000 I	1.56772
95.00	1.330	.000 I	1.36423
100.00	1.330	.000 I	1.23165
105.00	1.330	.000 I	1.14852
110.00	1.330	.000 I	1.08619
115.00	1.330	.000 I	1.06764
120.00	1.330	.000 I	1.09373
125.00	1.330	.000 I	1.13248
130.00	1.330	.000 I	1.19065
135.00	1.330	.000 I	1.30211
140.00	1.330	.000 I	1.42800
145.00	1.330	.000 I	1.54782
150.00	1.330	.000 I	1.75570
155.00	1.330	.000 I	2.01904
160.00	1.330	.000 I	2.05120
165.00	1.330	.000 I	1.75837
170.00	1.330	.000 I	1.51055
175.00	1.330	.000 I	1.57733
180.00	1.330	.000 I	1.68700

where

$$A_k(\eta, \omega) = \frac{4\omega}{nb} , R^2 \cdot \sum_{j=1}^{b} \cos^{-1}\left(\frac{\cos\omega - \cos\eta \cos\gamma_{k,j}}{\sin\eta \sin\gamma_{k,j}}\right) \sin\gamma_{k,j}$$

and

$$\theta_k = \eta - \omega\left(\frac{n - 2k + 1}{n}\right) , \quad \gamma_{k,j} = \eta - \frac{\omega}{n}\left(n - 2k + 3 - \frac{2j + 1}{b}\right) .$$

The angular apertures are divided into n strips, which are, in turn, subdivided into b substrips. Table 3 was produced from the computer program using an aperture of $2\omega = 6$ degrees, an observation angle η, $n = 5$, $b = 4$, and with the Mie program ($\alpha = 10.0$, $m = 1.33$). Figure 3 shows implicitly the aperture smoothing effect for these values (cf. Atlas for $\alpha = 10$).

More applications for the Mie program will be developed in the future. These will include a further analysis of the smoothing effects of both the polydispersions and the aperture. Other computer programs to be developed will be concerned with

1. Zodiacal models developed in full

2. Contaminant cloud models developed in detail to predict distributions and intensities

3. Problems associated with nonspherical particles.

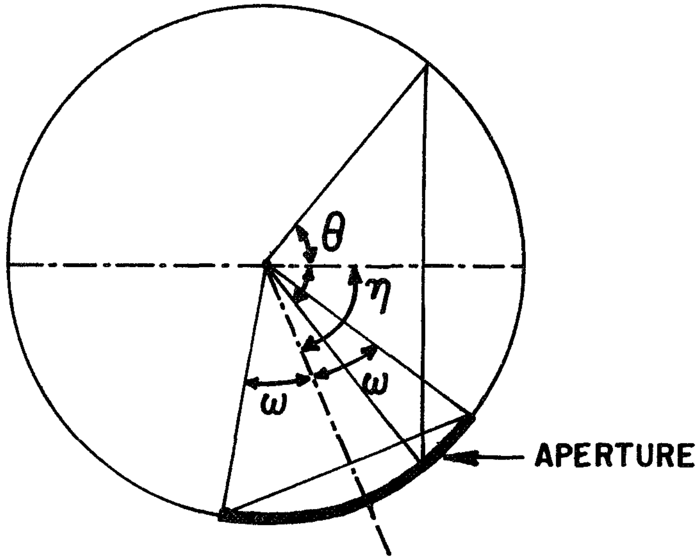


Figure 2. Lens aperture.

TABLE 3. EFFECT OF APERTURE WITH CHANGING ANGLE OF OBSERVATION

θ (deg)	Total Intensity (6 deg)	Total Intensity (0 deg)
150	20.1	20.0
151	15.2	14.0
152	12.0	10.0
153	10.9	8.1
154	12.1	9.0
155	15.5	12.3
156	20.7	17.8

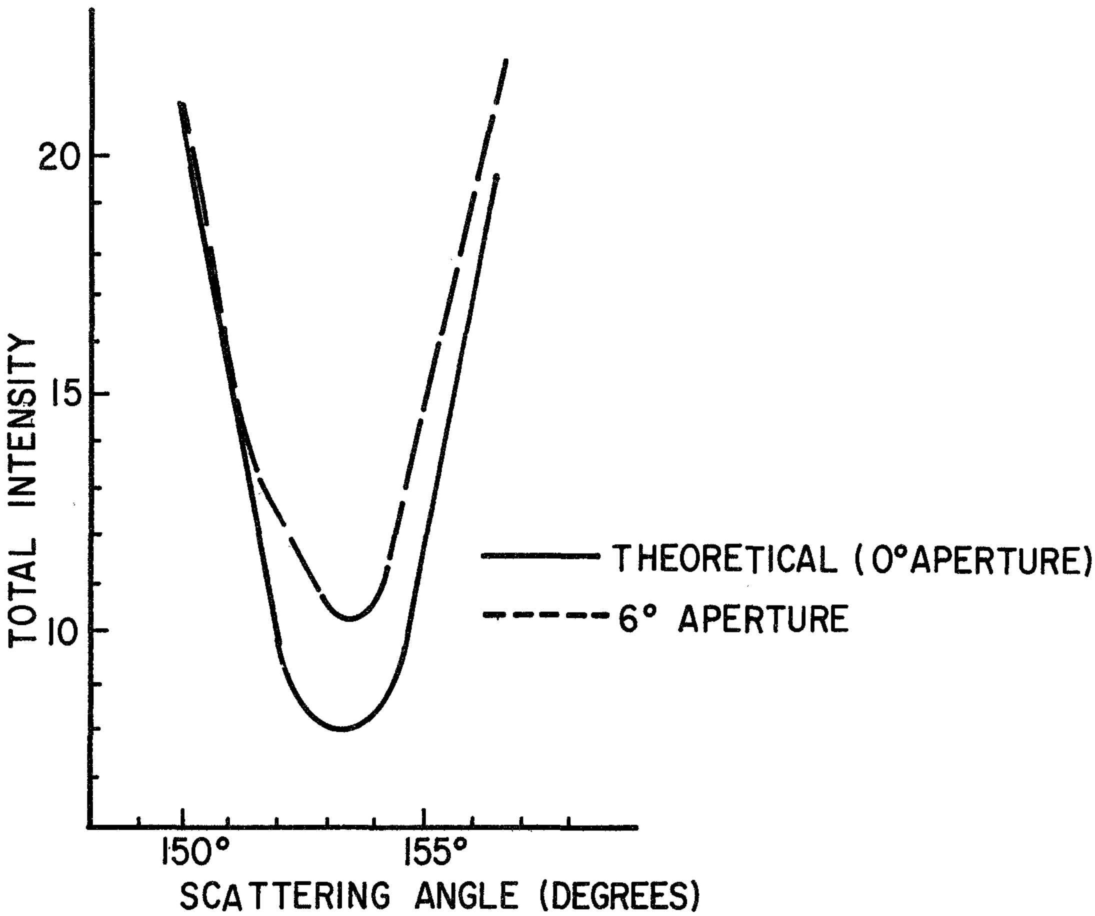


Figure 3. Intensity comparison of a 6-degree aperture with that of a 0-degree aperture.

APPENDIX A

MIE COMPUTER PROGRAMS

File Tape on Univac 1108

A more efficient use of computer time has been realized by filing large subroutines on tape for use in the Univac 1108 computer. Then, the calling program (e.g., an integration), can have a minimum execution time with no computer printout. The resultant control cards which are used for filing the Mie program on magnetic tape are given in Program List 1 where MIESCA is the Run Identification, MIE is the filed subroutine, MIETAP is the name of the newly formed tape, 000000 is the identification (ID) number, and 5 is the maximum number of minutes of computer time.

PROGRAM LIST 1. FILE SUBROUTINE MIE ON MIETAP

```
@RUN,A MIESCA,800080,NADINEBIN219,5
@ASG,T MIETAP,T,32024
@MSG,W PLEASE PUT WRITE RING IN TAPE 32024
@FOR,IS MIE
      SUBROUTINE MIE(XTH1,EM,ALT,XI1,XI2,X12,POLAR)
      DIMENSION EM(2),BET(2),XJNB(2),XJNBP(2),AUS3(2),RBSB(2),RBSBP(2)
     1,RHNK(2),RHNKP(2),P1(2),P2(2),P3(2),XNUMA(2),XNUMB(2),XDENA(2),
     1XDENB(2),AN(2),BN(2),XTH(37),PN(37),PNP(37),S1NR(37),S1NI(37),
     1S2NR(37),S2NI(37),S1R(37),S2R(37),S1I(37),S2I(37),XX(37),
     1AAUKS(37),AAUX(37)
      COMMON N,XN,ALP,BET,AUX1,AUX2,AUX3,AUX4,AUX5(2),AUX6(2),AUX,
     1AUKS,VX
      I=1
      ALP=ALT
      XTH(1)=XTH1
      PI=3.1415927
      DEG=PI/180.
   52 CALL ZXR(EM,ALP,BET,1)
   13 M=1
      XTH(I)=XTH(I)*DEG
   56 S1R(I)=0.
      S1I(I)=0.
      S2R(I)=0.
   11 S2I(I)=0.
      N=1
      AEXT=0.
      ASCAT=0.
   12 XN=FLOAT(N)
      CALL RECJ(XJN,XJNP,XJN1,XJN1P,XJNB,XJNBP,AUS1,AUS2,AUS3)
      AUX1=XJN
      AUX2=XJN1
      AUX3=AUS1
      AUX4=AUS2
      CALL ZEQUA(AUX5,XJNB)
      CALL ZEQUA(AUX6,AUS3)
      RBS=ALP*XJN
      RBSP=XJN+ALP*XJNP
      CALL ZMULT(BET,XJNB,RBSB)
      CALL ZMULT(BET,XJNBP,P1)
      CALL ZADD(XJNB,P1,RBSBP)
      RHNK(1)=RBS
      RHNK(2)=(-1.)**N*XJN1*ALP
      RHNKP(1)=RBSP
      RHNKP(2)=(-1.)**N*XJN1P*ALP+(-1)**N*XJN1
      CA=((XN+XN+1.)*(-1)**N)/(XN*XN+XN)
      CB=-CA
      CALL ZXR(RBSBP,RBS,P1,1)
      CALL ZXR(RBSB,RBSP,P2,1)
      CALL ZMULT(EM,P2,P3)
      CALL ZSUB(P1,P3,XNUMA)
      CALL ZMULT(EM,P1,P3)
      CALL ZSUB(P3,P2,XNUMB)
      CALL ZMULT(RHNK,RBSBP,P1)
      CALL ZMULT(RBSB,RHNKP,P2)
      CALL ZMULT(EM,P2,P3)
      CALL ZSUB(P1,P3,XDENA)
      CALL ZMULT(EM,P1,P3)
      CALL ZSUB(P3,P2,XDENB)
      CALL ZXI(XNUMA,CA,P1,1)
      CALL ZDIV(P1,XDENA,AN)
```

```
      CALL ZXI(XNUMB,CB,P1,1)
      CALL ZDIV(P1,XDENB,BN)
   17 CXN=XN*(XN+1.)*(-1.)**N
      AEXN=CXN*(AN(2)-BN(2))
      AEXT=AEXT+AEXN
      ABSAN=ZMOD(AN)
      ABSBN=ZMOD(BN)
      CKN=(XN*XN*(XN+1.)*(XN+1.))/(XN+XN+1.)
      ASCN=CKN*(ABSAN*ABSAN+ABSBN*ABSBN)
      ASCAT=ASCAT+ASCN
      IF(ABSAN-.0000001)21,21,23
   21 IF(ABSBN-.0000001)22,22,23
   22 IF(XN-ALP)23,23,25
   23 CONTINUE
   18 CONTINUE
      VX=-COS(XTH(I))
      IF(N-1)41,41,42
   42 AUKS=AAUKS(I)
      AUX=AAUX(I)
   41 CALL RECP(XLNP,XLNPP,AUS)
      PN(I)=XLNP
      PNP(I)=XLNPP
      AAUKS(I)=PN(I)
      AAUX(I)=AUS
      XX(I)=XN*(XN+1.)*PN(I)-VX*PNP(I)
      S1NR(I)=AN(1)*PNP(I)+BN(1)*XX(I)
      S1NI(I)=AN(2)*PNP(I)+BN(2)*XX(I)
      S2NR(I)=BN(1)*PNP(I)+AN(1)*XX(I)
      S2NI(I)=BN(2)*PNP(I)+AN(2)*XX(I)
      S1R(I)=S1R(I)+S1NR(I)
      S1I(I)=S1I(I)+S1NI(I)
      S2R(I)=S2R(I)+S2NR(I)
  100 S2I(I)=S2I(I)+S2NI(I)
  101 N=N+1
      GO TO 12
   25 ASCAT=ASCAT*2./(ALP*ALP)
      AEXT=AEXT*2./(ALP*ALP)
      AABS=AEXT-ASCAT
      XI1=S1R(I)*S1R(I)+S1I(I)*S1I(I)
      XI2=S2R(I)*S2R(I)+S2I(I)*S2I(I)
  116 POLAR=(XI1-XI2)/(XI1+XI2)
      X12=XI1+XI2
      RETURN
      END
@FOR,IS RECP
      SUBROUTINE RECP(PN,PNP,AUS)
      DIMENSION BET(2)
      COMMON N,XN,ALP,BET,AUX1,AUX2,AUX3,AUX4,AUX5(2),AUX6(2),AUX,
     1AUKS,VX
      IF(N-1)1,1,2
    1 PN=VX
      PNP=1.
      AUS=((2.*XN+1.)*PN*VX-XN)/(XN+1.)
      GO TO 3
    2 PN=AUX
      IF(ABS(VX-1.000000)-0.000001)21,21,22
   22 IF(ABS(VX+1.000000)-0.000001)21,21,23
   21 PNP=(XN*XN+XN)*PN/(VX+VX)
      GO TO 24
   23 PNP=(-XN*VX*PN+XN*AUKS)/(1.-VX*VX)
   24 AUS=((2.*XN+1.)*VX*PN-XN*AUKS)/(XN+1.)
```

```
    3 RETURN
      END
@FOR,IS RECJ
      SUBROUTINE RECJ(XJN,XJNP,XJN1,XJN1P,XJNB,XJNBP,AUS1,AUS2,AUS3)
      DIMENSION XJNB(2),XJNBP(2),XJOB(2),XJ1B(2),BET(2),
     1XJ2B(2),P1(2),P2(2),Q1(2),Q2(2),AUS3(2),SINB(2),COSB(2)
      COMMON N,XN,ALP,BET,AUX1,AUX2,AUX3,AUX4,AUX5(2),AUX6(2),AUX,
     1AUKS,VX
      IF(N-1)1,1,2
    1 XJO=SIN(ALP)/ALP
      XJ1=XJO/ALP-COS(ALP)/ALP
      XJN=XJ1
      XJM1=XJO/ALP-XJ1
      XJM2=(-XJM1)/ALP-XJO
      XJN1=XJM2
      XJ2=3.*XJ1/ALP-XJO
      XJNP=(XJO-2.*XJ2)/3.
      XJM3=(-3.)*XJM2/ALP-XJM1
      XJN1P=((-2.)*XJM3+XJM1)/(-3.)
      CALL ZSC(BET,SINB,COSB,1)
      CALL ZDIV(SINB,BET,XJOB)
      CALL ZDIV(XJOB,BET,Q1)
      CALL ZSC(BET,SINB,COSB,2)
      CALL ZDIV(COSB,BET,Q2)
      CALL ZSUB(Q1,Q2,XJ1B)
      CALL ZEQUA(XJNB,XJ1B)
      CALL ZXR(XJ1B,3.,P1,1)
      CALL ZDIV(P1,BET,Q1)
      CALL ZSUB(Q1,XJOB,XJ2B)
      CALL ZXR(XJ2B,2.,P1,1)
      CALL ZSUB(XJOB,P1,P2)
      CALL ZXR(P2,3.,XJNBP,2)
      AUS1=XJ2
      AUS2=XJM3
      CALL ZEQUA(AUS3,XJ2B)
      GO TO 3
    2 XJN=AUX3
      XJN1=AUX4
      AUS1=(2.*XN+1.)*AUX3/ALP-AUX1
      XJNP=(XN*AUX1-(XN+1)*AUS1)/(2.*XN+1.)
      AUS2=AUX4*(-2.*(XN+1.)+1.)/ALP-AUX2
      XJN1P=(-(XN+1.)*AUS2+XN*AUX2)/(-2.*(XN+1.)+1.)
      CALL ZEQUA(XJNB,AUX6)
      C=2.*XN+1.
      CALL ZXR(AUX6,C,P1,1)
      CALL ZDIV(P1,BET,Q1)
      CALL ZSUB(Q1,AUX5,AUS3)
      CALL ZXR(AUX5,XN,P1,1)
      CALL ZXR(AUS3,XN+1.,P2,1)
      CALL ZSUB(P1,P2,P1)
      CALL ZXR(P1,C,XJNBP,2)
    3 RETURN
      END
@FOR,IS ZDIV
      SUBROUTINE ZDIV(A,B,C)
      DIMENSION A(2),B(2),C(2),BSTAR(2)
      CALL ZCONJ(B,BSTAR)
      CALL ZMULT(A,BSTAR,C)
      C(1)=C(1)/(ZMOD(B)*ZMOD(B))
      C(2)=C(2)/(ZMOD(B)*ZMOD(B))
      RETURN
```

```
      END
@FOR,IS ZSUB
      SUBROUTINE ZSUB(A,B,C)
      DIMENSION A(2),B(2),C(2)
      C(1)=A(1)-B(1)
      C(2)=A(2)-B(2)
      RETURN
      END
@FOR,IS ZADD
      SUBROUTINE ZADD(A,B,C)
      DIMENSION A(2),B(2),C(2)
      C(1)=A(1)+B(1)
      C(2)=A(2)+B(2)
      RETURN
      END
@FOR,IS ZMUL
      SUBROUTINE ZMULT(A,B,C)
      DIMENSION A(2),B(2),C(2)
      C(1)=A(1)*B(1)-A(2)*B(2)
      C(2)=A(1)*B(2)+A(2)*B(1)
      RETURN
      END
@FOR,IS ZCON
      SUBROUTINE ZCONJ(A,ASTAR)
      DIMENSION A(2),ASTAR(2)
      ASTAR(1)=A(1)
      ASTAR(2)=-A(2)
      RETURN
      END
@FOR,IS ZMOD
      FUNCTION ZMOD(A)
      DIMENSION A(2)
      ZMOD=SQRT(A(1)*A(1)+A(2)*A(2))
      RETURN
      END
@FOR,IS RTHX
      SUBROUTINE RTHXY(RTH,A)
      DIMENSION RTH(2),A(2)
      A(1)=RTH(1)*COS(RTH(2))
      A(2)=RTH(1)*SIN(RTH(2))
      RETURN
      END
@FOR,IS ZXR
      SUBROUTINE ZXR(A,R,C,N)
      DIMENSION A(2),C(2)
      GO TO(1,2),N
    1 C(1)=A(1)*R
      C(2)=A(2)*R
      GO TO 3
    2 C(1)=A(1)/R
      C(2)=A(2)/R
    3 RETURN
      END
@FOR,IS ZXI
      SUBROUTINE ZXI(A,XI,C,N)
      DIMENSION A(2),C(2)
      GO TO(1,2),N
    1 C(1)=-A(2)*XI
      C(2)=A(1)*XI
      GO TO 3
    2 C(1)=A(2)/XI
```

PROGRAM LIST 1. (Concluded)

```
      C(2)=-A(1)/XI
    3 RETURN
      END
@FOR,IS ZSC
      SUBROUTINE ZSC(A,SINA,COSA,N)
      DIMENSION A(2),SINA(2),COSA(2),C1(2),C2(2),RTH1(2),RTH2(2)
      RTH1(1)=1.
      RTH1(2)=A(1)
      RTH2(1)=1.
      RTH2(2)=-A(1)
      ER1=EXP(-A(2))
      ER2=EXP(A(2))
      CALL RTHXY(RTH1,C1)
      CALL RTHXY(RTH2,C2)
      CALL ZXR(C1,ER1,C1,1)
      CALL ZXR(C2,ER2,C2,1)
      GO TO (1,2),N
    1 CALL ZSUB(C1,C2,C1)
      CALL ZXI(C1,2.,SINA,2)
      GO TO 3
    2 CALL ZADD(C1,C2,C1)
      CALL ZXR(C1,2.,COSA,2)
    3 RETURN
      END
@FOR,IS ZEQU
      SUBROUTINE ZEQUA(A,B)
      DIMENSION A(2),B(2)
      A(1)=B(1)
      A(2)=B(2)
      RETURN
      END
@MAP,IS A,B
 IN MIE
@COPOUT  TPF$.,MIETAP.
@REWIND MIETAP.
@ERS
@COPIN  MIETAP.,TPF$.,
@REWIND MIETAP.
@FIN
@FIN
```

For calling MIE out of the tape file [4], the following sequence is used, with tape label 32024, Run ID, and calling program DRYRUN.

PROGRAM LIST 2. CALL UP FILED MIETAP IN DRYRUN

```
@RUN,A DRYRUN,000000,NAME
@ASG,T MIETAP,T,32024
@COPIN MIETAP.,TPF$.
@REWIND MIETAP
@FOR,IS DRYRUN
    ...
    Main program
    ...
```

```
    CALL MIE (XTH1,EM,ALT,XI1,XI2,X12,POLAR)
    ...
@MAP,IS A,B
@X
    IN DRYRUN
@XQT
    Data
@FIN
```

SC 4020 Plots

The plots used in the Atlas were done by the SC 4020 plotter called up in the Univac 1108 MIESCA program. For the example in the text with $\alpha = 5.0$, we used Program List 3 for the main program. (Subroutines are the same as for Program List 1.)

The dimension statements and the data statements supplying the abscissa and ordinate labels are labeled A.

The first card needed for addressing the plotter is the CALL IDENT (CAMRAS), labeled B. Depending on the number in parentheses, the output will be on paper or microfilm:

CAMRAS = 35 film
CAMRAS = 9 paper
CAMRAS = 1 film and paper.

The values for the array to be plotted are generated in the program. The CALL SMXYV(MX,MY) is inserted (labeled E), where MX, MY are fixed-point integers which designate whether the linear or nonlinear (logarithmic) mode is to be used.

If [4]	$MX \neq 0$, $MY \neq 0$	log in X, log in Y
	$MX \neq 0$, $MY = 0$	log in X, linear in Y
	$MX = 0$, $MY \neq 0$	linear in X, log in Y
	$MX = 0$, $MY = 0$	linear in X, linear in Y .

SMXYV must be called each time before there is a change of mode. Now, QUKLOG and QUIK3V can be labeled F. The general forms for calling the subroutines are:

```
CALL QUIK3V(L,ISYM,FLDX,FLDY,NP,X,Y)
CALL QUIK3L(L,XL,XR,YB,YT,ISYM,FLDX,FLDY,NP,X,Y)
CALL QUKLOG(L,XL,XR,YB,YT,ISYM,FLDX,FLDY,NP,X,Y)
```

PROGRAM LIST 3. SC 4020 ADAPTATION OF MIESCA (MAIN PROGRAM ONLY)

```
@RUN,A MIESCA,400980,GARYMIESCAT1,2
@FOR,IS GARYGA
C  THIS PROGRAM COMPUTES THE MIE SOLUTIONS FOR ARBITRARY PHYSICAL
C  PARAMETERS.
C
C  DESCRIPTION OF INPUT PARAMETERS -
C    NI     ---- TOTAL NUMBER OF SETS OF INPUT PHYSICAL PARAMETERS
C    EACH SET OF INPUT PHYSICAL PARAMETERS CONTAINS -
C    ALP    ---- THE PARAMETER ALPHA
C    EM     ---- THE COMPLEX REFRACTIVE INDEX
C    EACH SET OF INPUT PHYSICAL PARAMETERS IS FOLLOWED BY A SET OF
C    OPTION CODES -
C    NOPT1 ---- ONE DIGIT NUMBER INDICATING HOW THE INTENSITY
C               FUNCTIONS ARE TO BE COMPUTED
C               1 - THE INTENSITY FUNCTIONS ARE COMPUTED FOR
C                   SCATTERING ANGLES VARYING FROM 0 TO 180 DEGREES
C                   AT 5 DEGREE INTERVALS
C               2 - THE INTENSITY FUNCTIONS ARE COMPUTED FOR SPECIFIED
C                   SCATTERING ANGLES
C    NOPT2 ---- ONE DIGIT NUMBER INDICATING WHETHER THE AMPLITUDE
C               FUNCTIONS ARE TO BE PRINTED
C               1 - YES
C               2 - NO
C    NOPT3 ---- ONE DIGIT NUMBER INDICATING WHETHER THE INTENSITY
C               FUNCTIONS ARE TO BE COMPUTED
C               1 - YES
C               2 - NO
C    NOTE - WHEN NOPT3=2, IT OVERRIDES NOPT1. NOPT1 MUST BE SET EQUAL
C    TO 1 IN THIS CASE
C    WHEN NOPT1=2, ADDITIONAL PARAMETERS MUST FOLLOW THE SAME SET OF
C    INPUT PHYSICAL PARAMETERS -
C    M      ---- NUMBER OF SPECIFIED SCATTERING ANGLES
C    NOTE - M MUST BE SMALLER THAN OR EQUAL TO 37
C    XTH    ---- A VECTOR OF LENGHTH M WITH XTH(I) EQUAL TO THE I-TH
C               SPECIFIED SCATTERING ANGLE EXPRESSED IN DEGREES
C
C  DESCRIPTION OF OUTPUT QUANTITIES -
C    AN,BN ---- THE COMPLEX AMPLITUDE FUNCTIONS
C    XI1,XI2 -- THE INTENSITY FUNCTIONS
C    AEXT  ---- THE EFFICIENCY FACTOR FOR TOTAL EXTINCTION
C    ASCAT ---- THE EFFICIENCY FACTOR FOR SCATTERING
C    AABS  ---- THE EFFICIENCY FACTOR FOR ABSORPTION
C    POLAR ---- DEGREE OF POLARIZATION
C    ALL OUTPUT QUANTITIES ARE PRINTED UNLESS THEY HAVE BEEN RULED OUT
C    BY THE OPTION CODES
C
C  REMARKS -
C    IN ALL COMPUTATIONS OF INFINITE SERIES, CONVERGENCE IS ASSUMED
C    TO HAVE BEEN ACHIEVED WHEN BOTH AMPLITUDE FUNCTIONS ARE SMALLER
C    THAN OR EQUAL TO 1.0E-07, AND THE NUMBER OF TERMS COMPUTED
C    EXCEEDS THE VALUE OF ALP.
C
*     DIMENSION EM(2),BET(2),XJNB(2),XJNBP(2),AUS3(2),RBSB(2),RBSBP(2)
     1,RHNK(2),RHNKP(2),P1(2),P2(2),P3(2),XNUMA(2),XNUMB(2),XDENA(2),
     1XDENB(2),AN(2),BN(2),XTH(37),PN(37),PNP(37),S1NR(37),S1NI(37),
     1S2NR(37),S2NI(37),S1R(37),S2R(37),S1I(37),S2I(37),XX(37),
     1AAUKS(37),AAUX(37)
      DIMENSION ARRAY1(2000),BRRAY1(2000),CRRAY1(2000),DRRAY1(2000),
     1ARRAY2(2000),BRRAY2(2000),CRRAY2(2000),DRRAY2(2000)
A     DIMENSION AFDX(12),AFDY(12),BFDY(12),CFDY(12),DFDY(12)
```

```
A      DATA AFDX/72H THETA DEGREES
      1
       DATA AFDY/72H1000 TIMES INTENSITY 1 ( * ) AND 1000 TIMES INTENSITY
      1 2 ( + )
       DATA BFDY/72H1000 TIMES INTENSITY 1 ( • ) AND 1000 TIMES INTENSITY
      1 2 ( + )
       DATA CFDY/72H 1000 TIMES ( INTENSITY 1 + INTENSITY 2 )
      1
       DATA DFDY/72H POLARIZATION
      1
       COMMON N,XN,ALP,BET,AUX1,AUX2,AUX3,AUX4,AUX5(2),AUX6(2),AUX,
      1AUKS,VX
       COMMON V(4),ISTORE(901)
B      CALL IDENT(1)
     1 FORMAT(I4)
     2 FORMAT(3F10.3,5X,3I1)
     3 FORMAT(1H0,'REFRACTIVE INDEX   RE(M) =',F7.3,3X,'IM(M) =',F7.3,
      1/,1H ,'PARAMETER ALPHA =',F7.2,/)
     4 FORMAT(F8.3)
     5 FORMAT(1H0,19X,'AMPLITUDE FUNCTIONS',//,3X,'N',7X,'RE(AN)',
      18X,'IM(AN)',8X,'RE(BN)',8X,'IM(BN)',/)
     6 FORMAT(I4,4F14.6)
     7 FORMAT(1H0,9X,'EFFICIENCY FACTOR FOR EXTINCTION QEXT  =',
      1F8.5,//,10X,'EFFICIENCY FACTOR FOR SCATTERING QSCAT =',
      1F8.5,//,10X,'EFFICIENCY FACTOR FOR ABSORPTION QABS  =',
      1F8.5,/)
     8 FORMAT(1H0,'SCAT ANGLE',8X,'INTENSITY FUNCTIONS',4X,
      1'DEGREE OF POLARIZATION',/,1H0,'THETA(DEG)',9X,
      1'I1',13X,'I2',14X,'P',13X,'I1+I2',/)
     9 FORMAT(1H ,I6,4X,2F15.6,4X,F9.6,F15.6)
    10 FORMAT(1H ,F7.3,3X,2F15.6,4X,F9.6,F15.6)
  1010 FORMAT(1H1,'DATA SET NO.',I3,5X,'MIE SCATTERING')
       PI=3.1415927
       DEG=PI/180.
       READ(5,1)NI
       NN=1
    50 CONTINUE
       WRITE(6,1010)NN
C   50 READ(....ETC.
       READ(5,2)ALP,EM(1),EM(2),NOPT1,NOPT2,NOPT3
       WRITE(6,3)EM(1),EM(2),ALP
       GO TO(51,52),NOPT2
    51 WRITE(6,5)
    52 CALL ZXR(EM,ALP,BET,1)
       GO TO(13,14),NOPT1
*   13 M=37
       GO TO 53
    14 READ(5,1)M
    53 DO 11 I=1,M
       GO TO(54,55),NOPT1
*   54 XTH(I)=FLOAT(I-1)*5.*DEG
       GO TO 56
    55 READ(5,4)XTH(I)
       XTH(I)=XTH(I)*DEG
    56 S1R(I)=0.
       S1I(I)=0.
       S2R(I)=0.
    11 S2I(I)=0.
       N=1
       AEXT=0.
       ASCAT=0.
```

```
12 XN=FLOAT(N)
   CALL RECJ(XJN,XJNP,XJN1,XJN1P,XJNB,XJNBP,AUS1,AUS2,AUS3)
   AUX1=XJN
   AUX2=XJN1
   AUX3=AUS1
   AUX4=AUS2
   CALL ZEQUA(AUX5,XJNB)
   CALL ZEQUA(AUX6,AUS3)
   RBS=ALP*XJN
   RBSP=XJN+ALP*XJNP
   CALL ZMULT(BET,XJNB,RBSB)
   CALL ZMULT(BET,XJNBP,P1)
   CALL ZADD(XJNB,P1,RBSBP)
   RHNK(1)=RBS
   RHNK(2)=(-1.)**N*XJN1*ALP
   RHNKP(1)=RBSP
   RHNKP(2)=(-1.)**N*XJN1P*ALP+(-1)**N*XJN1
   CA=((XN+XN+1.)*(-1)**N)/(XN*XN+XN)
   CB=-CA
   CALL ZXR(RBSBP,RBS,P1,1)
   CALL ZXR(RBSB,RBSP,P2,1)
   CALL ZMULT(EM,P2,P3)
   CALL ZSUB(P1,P3,XNUMA)
   CALL ZMULT(EM,P1,P3)
   CALL ZSUB(P3,P2,XNUMB)
   CALL ZMULT(RHNK,RBSBP,P1)
   CALL ZMULT(RBSB,RHNKP,P2)
   CALL ZMULT(EM,P2,P3)
   CALL ZSUB(P1,P3,XDENA)
   CALL ZMULT(EM,P1,P3)
   CALL ZSUB(P3,P2,XDENB)
   CALL ZXI(XNUMA,CA,P1,1)
   CALL ZDIV(P1,XDENA,AN)
   CALL ZXI(XNUMB,CB,P1,1)
   CALL ZDIV(P1,XDENB,BN)
   GO TO(16,17),NOPT2
16 WRITE(6,6)N,AN(1),AN(2),BN(1),BN(2)
17 CXN=XN*(XN+1.)*(-1.)**N
   AEXN=CXN*(AN(2)-BN(2))
   AEXT=AEXT+AEXN
   ABSAN=ZMOD(AN)
   ABSBN=ZMOD(BN)
   CKN=(XN*XN*(XN+1.)*(XN+1.))/(XN+XN+1.)
   ASCN=CKN*(ABSAN*ABSAN+ABSBN*ABSBN)
   ASCAT=ASCAT+ASCN
   IF(ABSAN-.0000001)21,21,23
21 IF(ABSBN-.0000001)22,22,23
22 IF(XN-ALP)23,23,25
23 GO TO(18,101),NOPT3
18 DO 100 I=1,M
   VX=-COS(XTH(I))
   IF(N-1)41,41,42
42 AUKS=AAUKS(I)
   AUX=AAUX(I)
41 CALL RECP(XLNP,XLNPP,AUS)
   PN(I)=XLNP
   PNP(I)=XLNPP
   AAUKS(I)=PN(I)
   AAUX(I)=AUS
   XX(I)=XN*(XN+1.)*PN(I)-VX*PNP(I)
   SINR(I)=AN(1)*PNP(I)+BN(1)*XX(I)
```

```
        S1NI(I)=AN(2)*PNP(I)+BN(2)*XX(I)
        S2NR(I)=BN(1)*PNP(I)+AN(1)*XX(I)
        S2NI(I)=BN(2)*PNP(I)+AN(2)*XX(I)
        S1R(I)=S1R(I)+S1NR(I)
        S1I(I)=S1I(I)+S1NI(I)
        S2R(I)=S2R(I)+S2NR(I)
    100 S2I(I)=S2I(I)+S2NI(I)
    101 N=N+1
        GO TO 12
     25 ASCAT=ASCAT*2./(ALP*ALP)
        AEXT=AEXT*2./(ALP*ALP)
        AABS=AEXT-ASCAT
        WRITE(6,7)AEXT,ASCAT,AABS
        GO TO(103,104),NOPT3
    103 WRITE(6,8)
        DO 102 I=1,M
        XI1=S1R(I)*S1R(I)+S1I(I)*S1I(I)
        XI2=S2R(I)*S2R(I)+S2I(I)*S2I(I)
        IF(XI2)115,115,116
    115 XI2=.0000001
        WRITE(6,117)I
    117 FORMAT(1H ,6X,9HBRRAY 2 (,I5,32H) IS LESS THAN OR EQUAL TO ZERO.)
    116 POLAR=(XI1-XI2)/(XI1+XI2)
        GO TO(105,106),NOPT1
  * 105 NTH=(I-1)*5
        X12=XI1+XI2
        WRITE(6,9)NTH,XI1,XI2,POLAR,X12
        ARRAY1(I) = FLOAT(NTH)
        BRRAY1(I) = FLOAT(NTH)
        CRRAY1(I) = FLOAT(NTH)
        DRRAY1(I) = FLOAT(NTH)
  D     ARRAY2(I) = XI1
        BRRAY2(I) = XI2
        CRRAY2(I) = X12
        DRRAY2(I) = POLAR
        GO TO 102
    106 ANG=XTH(I)/DEG
        X12=XI1+XI2
        WRITE(6,10)ANG,XI1,XI2,POLAR,X12
        GO TO 102
    102 CONTINUE
        GO TO (1011,1012),NOPT1
   1011 CONTINUE
      E CALL SMXYV(0,1)
        CALL QUKLOG(-1,0.,180.,.1,1000.,1H*,AFDX,AFDY,-M,ARRAY1,ARRAY2)
F       CALL QUKLOG(0,0.,180.,.1,1000.,1H+,AFDX,BFDY,-M,BRRAY1,BRRAY2)
        CALL QUKLOG(-1,0.,180.,.1,1000.,1H*,AFDX,CFDY,-M,CRRAY1,CRRAY2)
      E CALL SMXYV(0,0)
        CALL QUIK3V(-1,1H*,AFDX,DFDY,-M,DRRAY1,DRRAY2)
   1012 CONTINUE
    104 NN=NN+1
        IF(NN-NI)50,50,110
G   110 CALL ENDJOB
        STOP
        END
```

where L	Fixed point integer which represents the number of graphs per frame (1, 2, or 3). If L is negative, the frame will be advanced, the grid for the first graph plotted, scaled, labeled, and the X and Y points will be plotted. If L is positive, the grid for the second or third is plotted, scaled, and labeled, and the X and Y points are plotted. If L is zero, the X and Y points are plotted using the previous grid. For example, suppose that the user wants three graphs on a frame. If two graphs are desired on the middle grid, then in four calls the L values, in order, would be -3, 3, 0, 3.
XL, XR	Floating point data values which represent the least and greatest X value respectively (for horizontal axis).
YB, YT	Floating point data values which represent the least and greatest Y value respectively (for vertical axis).
ISYM	Fixed point integer which contains the plotting symbol to be used. This may be a single Hollerith character or an integer identifying the desired symbol. The Univac 1108 and the SC-4020 do not have identical characters.
FLDX, FLDY	Fixed point integer arrays which contain Fieldata labels for the X and Y axes. FLDX and FLDY must each contain 12 words. The information to be centered must be left justified in the 72 character field. Remaining characters not used must be filled with blanks.
NP	Fixed point integer which represents the number of points to be plotted. <u>If NP is negative</u>, the plotted points will be connected by straight line segments.
X, Y	Floating point one dimensional arrays which contain X- and Y-coordinates of data to be plotted. X contains X-coordinates. Y contains Y-coordinates.

The very last call of SC 4020 routines must be the card CALL ENDJOB (G), or some of the last frame of the graphs might be lost.

The lines marked by an asterisk (*) are for the cards that can be altered to change the number of points plotted. For example, if the 5 in these cards (e.g., XTH(1) = FLOAT (I-1)* 5* DEG) is replaced by a 1 (as it was in the Atlas for $\alpha = 10 \rightarrow 100$), and M is changed to 181, then points will be plotted at 1 degree intervals of θ instead of 5 degree intervals. It should be remembered that all the 37's in the first dimension statement (with an asterisk) should be changed to 181's. The results of this program in printout for $\alpha = 5.0$ are found in Table 1 of the text. The SC 4020 plot for $\alpha = 5.0$ is in the Atlas.

APPENDIX B

NUMERICAL INTEGRATION METHODS USED

Gauss-Legendre Quadrature

Gauss' formula for integration [5] is:

$$\int_{-1}^{1} f(x)\,dx = \sum_{i=1}^{n} w_i f(x_i) + R_n$$

R_n is a remainder term, negligible for the applications in this report. The x_i's are the i^{th} zeros of Legendre polynomials, $P_n(x)$, and the w_i's are the weighting functions determined by

$$w_i = \frac{2}{(1 - x_i)^2} \left[P_n^1(x_i)\right]^2 ,$$

and are listed in Table 4, for $n = 16$.

TABLE 4. GAUSS-LEGENDRE WEIGHTS AND ABSCISSAS

$\pm x_i$				$n = 16$	w_i			
0.09501	25098	37637	440185		0.18945	06104	55068	496285
0.28160	35507	79258	913230		0.18260	34150	44923	588867
0.45801	67776	57227	386342		0.16915	65193	95002	538189
0.61787	62444	02643	748447		0.14959	59888	16576	732081
0.75540	44083	55003	033895		0.12462	89712	55533	872052
0.86563	12023	87831	743880		0.09515	85116	82492	784810
0.94457	50230	73232	576078		0.06225	35239	38647	892863
0.98940	09349	91649	932596		0.02715	24594	11754	094852

For an arbitrary interval (a, b) the general formula can be modified to give:

$$\int_a^b f(y)\,dy = \frac{b-a}{2} \sum_{i=1}^{n} w_i\, f(y_i) + R_n \qquad ,$$

where,

$$y_i = \left(\frac{b-a}{2}\right) x_i + \left(\frac{b+a}{2}\right)$$

This integration procedure was used in the aperture and distribution applications.

Gauss-Laguerre Quadrature

The Gauss-Laguerre equation for numerical integration is of the form

$$\int_0^\infty e^{-x} f(x)\,dx = \sum_{i=1}^{n} w_i\, f(x_i) \qquad ,$$

and is useful for integrations which do not have a finite upper limit. The equation is used in one distribution application with a semi-infinite interval and a distribution according to e^{-x}.

A table of w_i's and x_i's for $n = 15$ is presented in Table 5.

Trapezoidal Quadrature

The trapezoidal quadrature method was used as a check because it is not as accurate as the two aforementioned methods. The extended trapezoidal rule (without the remainder) is

$$\int_a^b f(x)dx = h\left[\frac{f(a)}{2} + f(a+1) + \ldots + f(b-1) + \frac{f(b)}{2}\right]$$

where (a, b) is the interval, $f(x)$ is the function, and h is the increment of integration [5].

TABLE 5. GAUSS-LAGUERRE WEIGHTS AND ABSCISSAS FOR n = 15 [5]

x_i	w_i	$w_i e^{x_i}$
0.093307812017	(- 1)2.18234885940	0.239578170311
0.492691740302	(- 1)3.42210177923	0.560100842793
1.215595412071	(- 1)2.63027577942	0.887008262919
2.269949526204	(- 1)1.26425818106	1.22366440215
3.667622721751	(- 2)4.02068649210	1.57444872163
5.425336627414	(- 3)8.56387780361	1.94475197653
7.565916226613	(- 3)1.21243614721	2.34150205664
10.120228568019	(- 4)1.11674392344	2.77404192683
13.130282482176	(- 6)6.45992676202	3.25564334640
16.654407708330	(- 7)2.22631690710	3.80631171423
20.776478899449	(- 9)4.22743038498	4.45847775384
25.623894226729	(-11)3.92189726704	5.27001778443
31.407519169754	(-13)1.45651526407	6.35956346973
38.530683306486	(-16)1.48302705111	8.03178763212
48.026085572686	(-20)1.60059490621	11.5277721009

APPENDIX C

MIE SCATTERING ATLAS

The following graphs, produced by the SC 4020 plotter, give

1. the scattering intensities (i_1 and i_2)

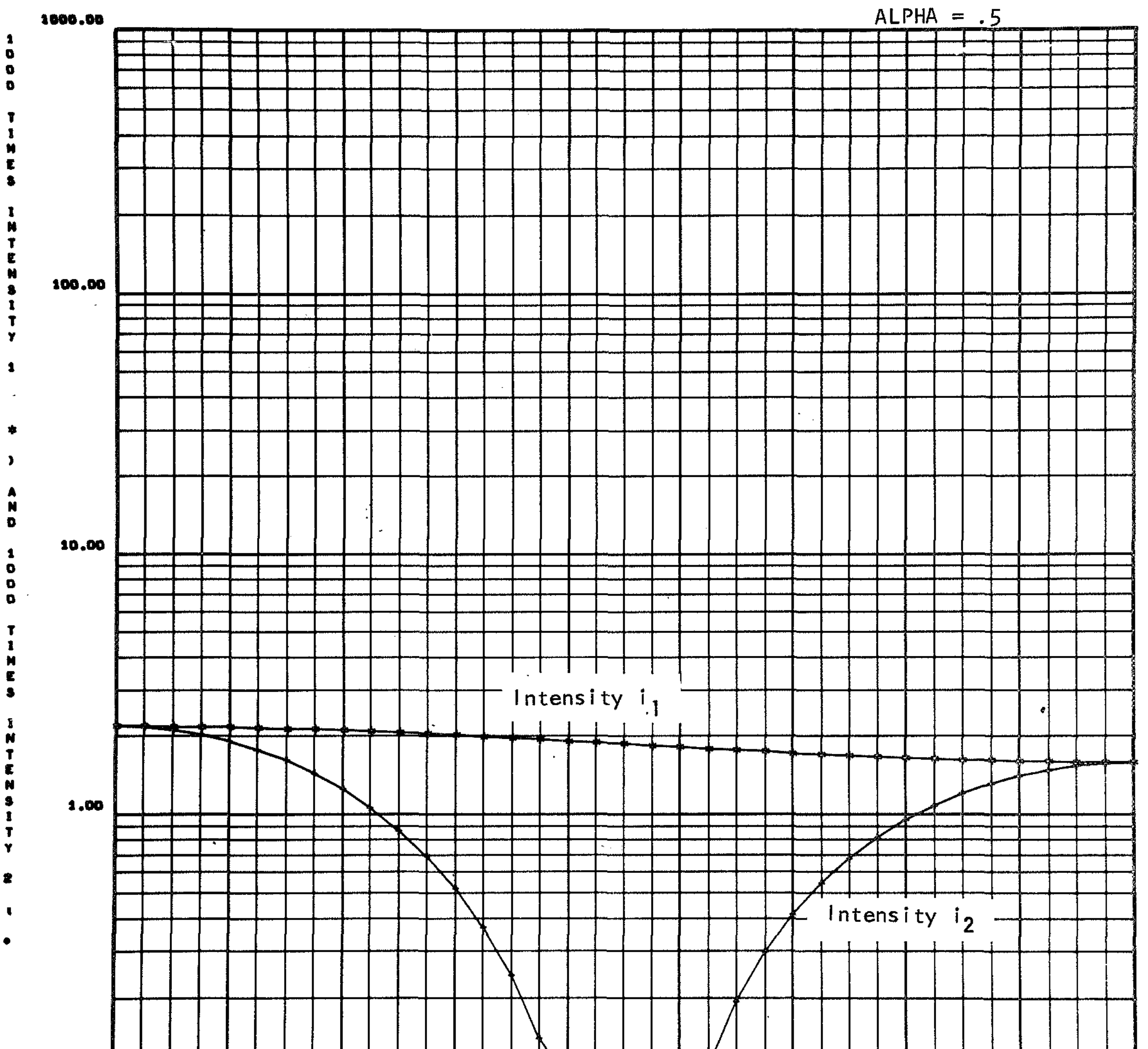
2. the total scattering intensity ($i_1 + i_2$)

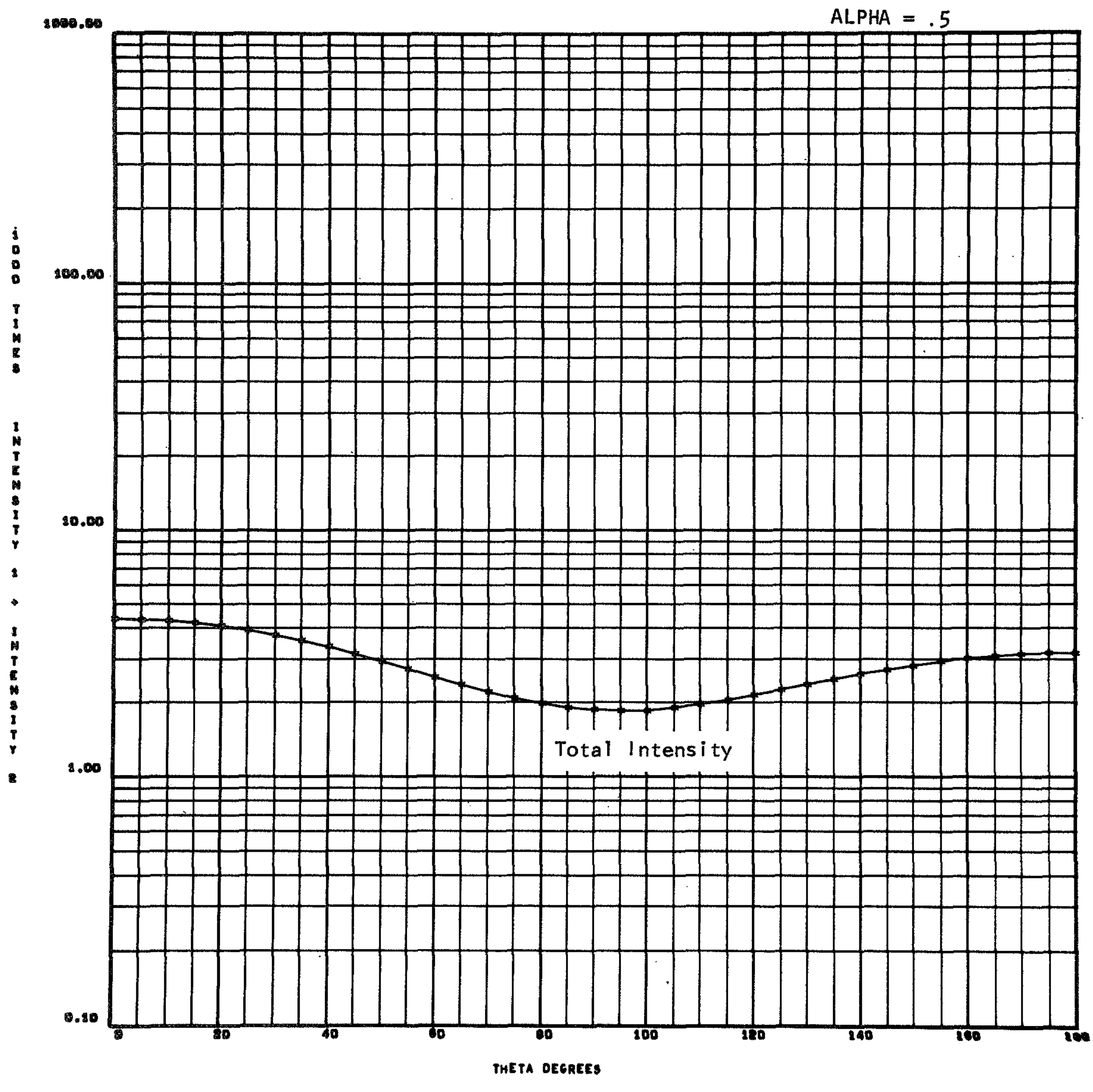
3. the polarization, $(i_1 - i_2)/(i_1 + i_2)$.

For α = 0.5, 0.9, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, and 10.0, the points are plotted for 5-degree intervals of the scattering angle. For α = 10.0, 20.0, 30.0, 40.0, 50.0, 60.0, 70.0, 80.0, 90.0, and 100.0, the points are plotted for intervals of 1 degree. The refractive index is m = 1.33.

At α = 0.5 the graphs have the form of Rayleigh Scattering. As α increases, the symmetry becomes more and more distorted and the complexity of the scattering diagram for large spheres is seen.

For the first set of graphs, α = 0.5, the meaning of each curve is explicitly indicated. For α = 0.5 and 0.9 the ordinate for the intensities has been multiplied by 10^3. The points for the two components of the intensity are labeled with an asterisk (*) for i_1 and a plus (+) for i_2.


ALPHA = .5
1000.00
100.00
10.00
1.00
0.10
1000 TIMES INTENSITY 1 (*) AND 1000 TIMES INTENSITY 2 (•)
0
20
40
60
80
100
120
140
160
180
THETA DEGREES
Intensity i_1
Intensity i_2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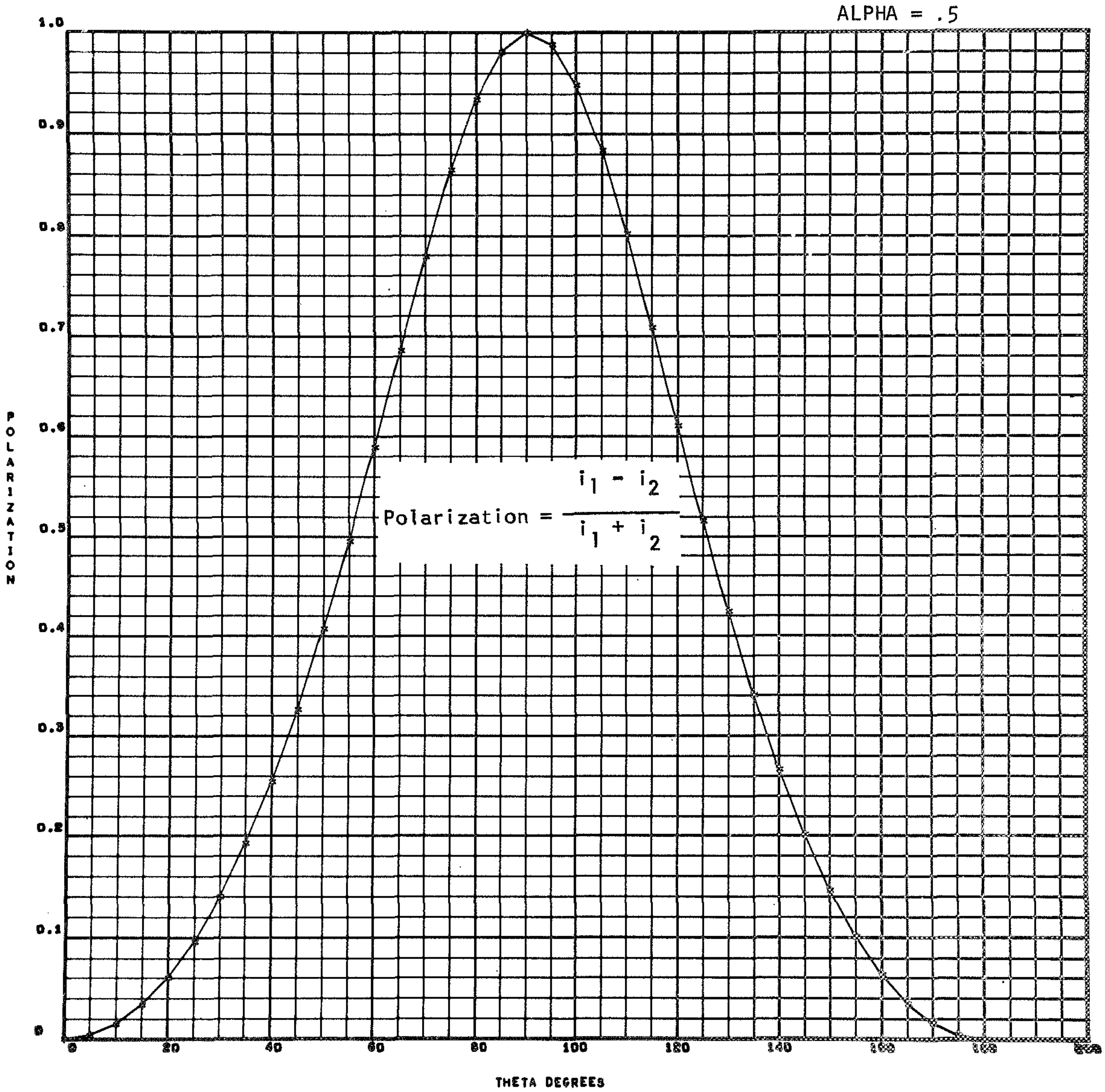
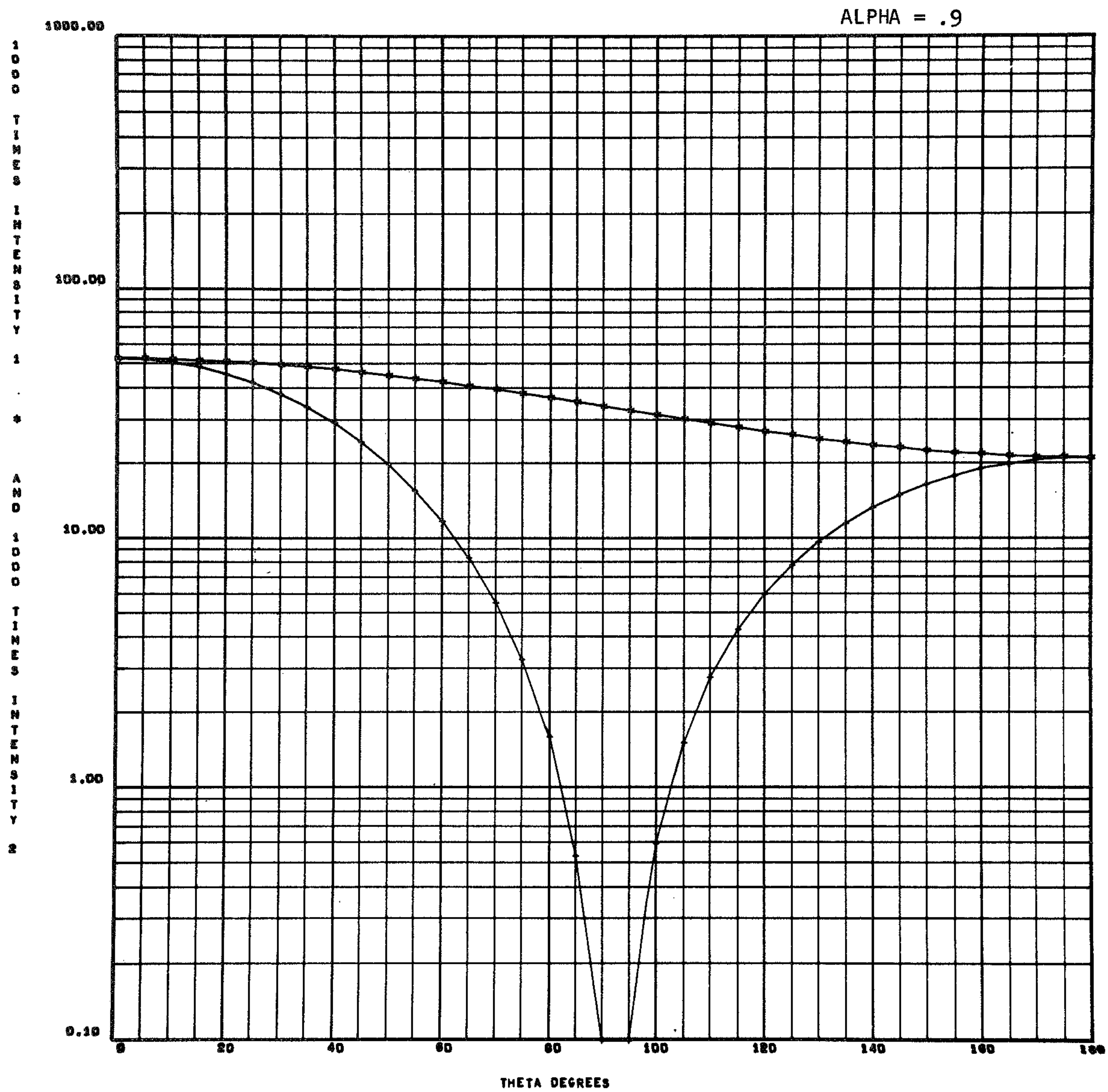
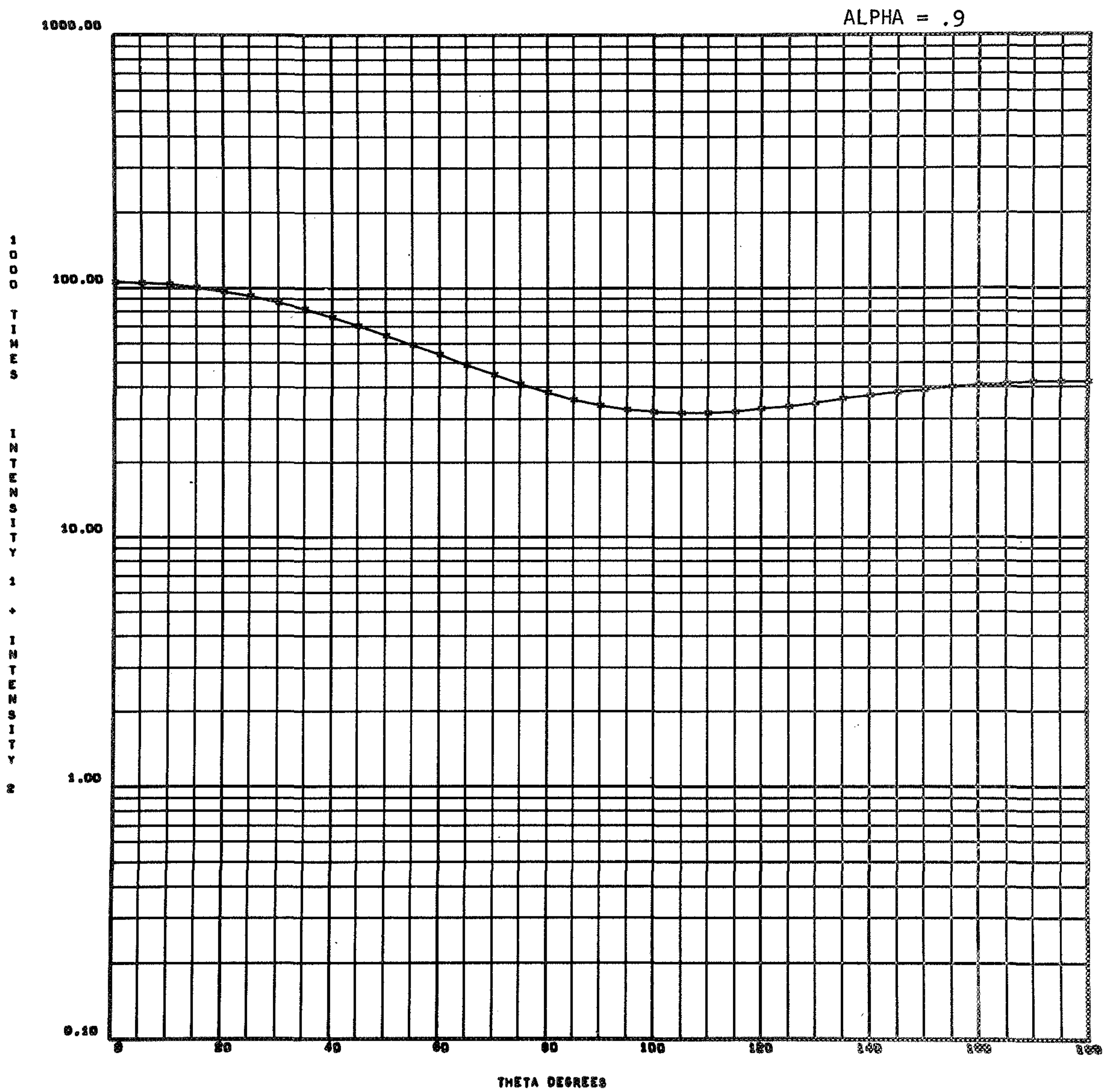
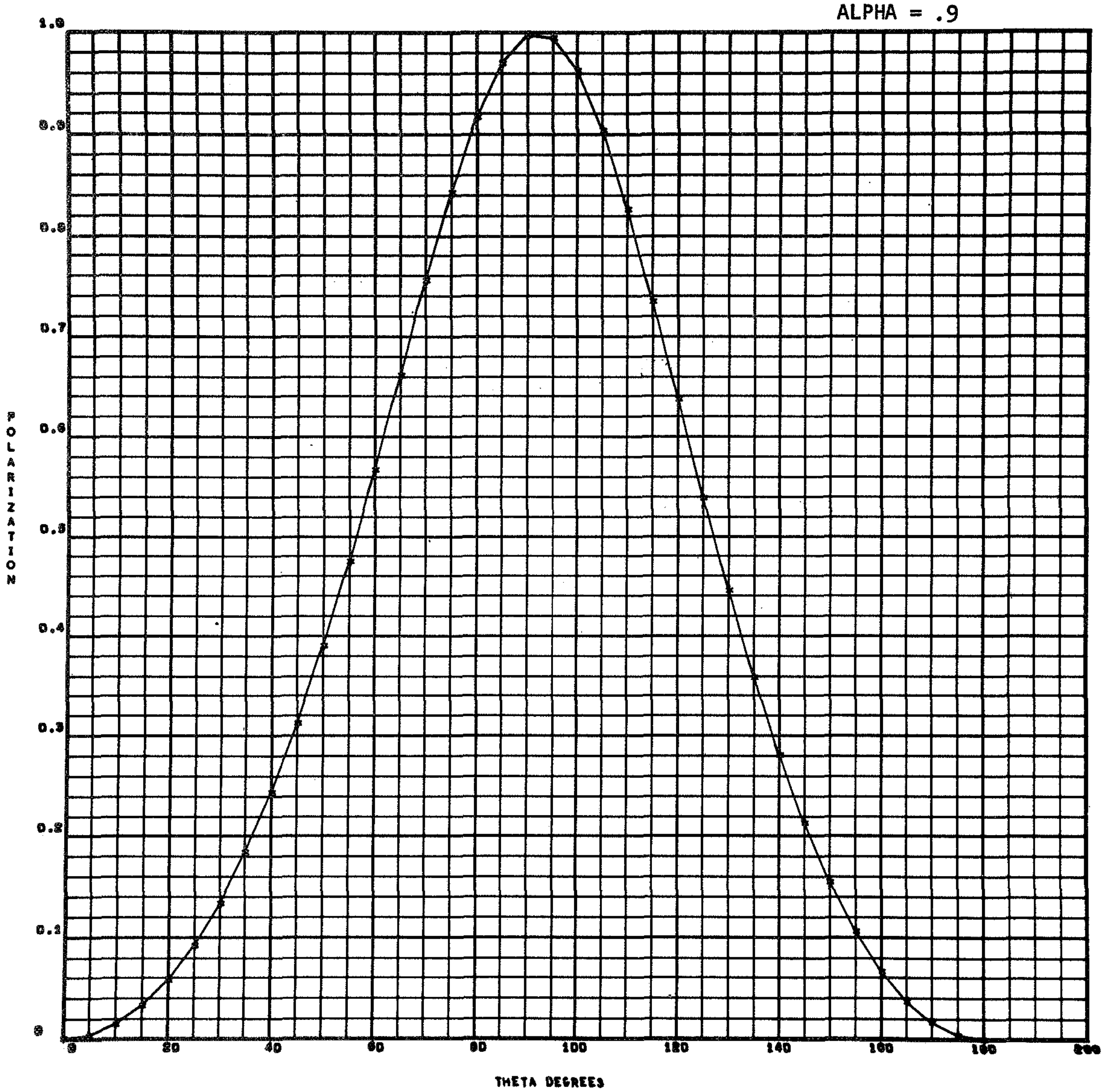
100.00
10.00
1.00
0.10
1000 TIMES INTENSITY 1 + INTENSITY 2
Total Intensity
0
20
40
60
80
100
120
140
160
180
THETA DEGREES


ALPHA = .5
$$\text{Polarization} = \frac{i_1 - i_2}{i_1 + i_2}$$
POLARIZATION
THETA DEGREES

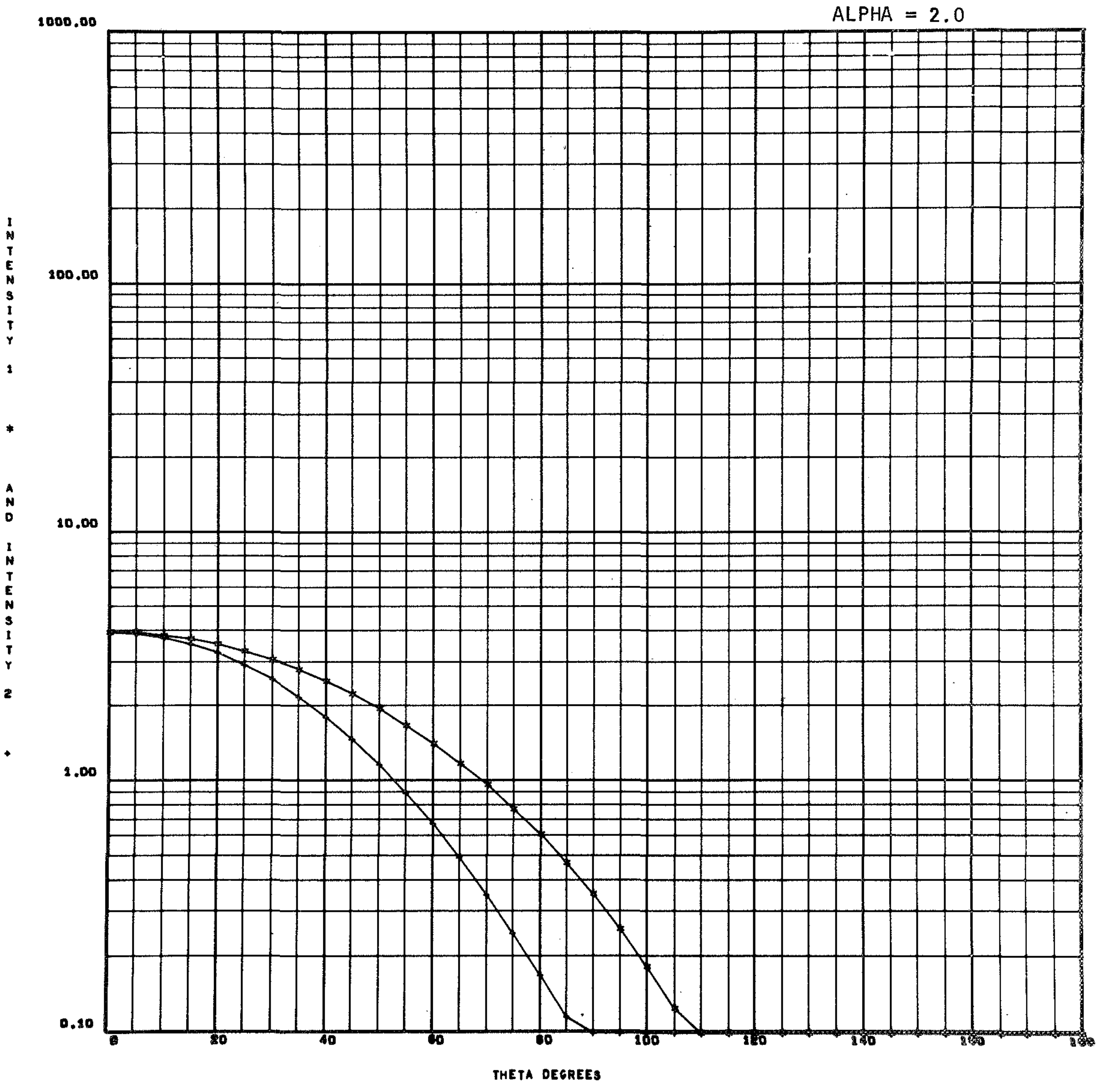

ALPHA = .9
1000.00
100.00
10.00
1.00
0.10
1000 TIMES INTENSITY 1 . * AND 1000 TIMES INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

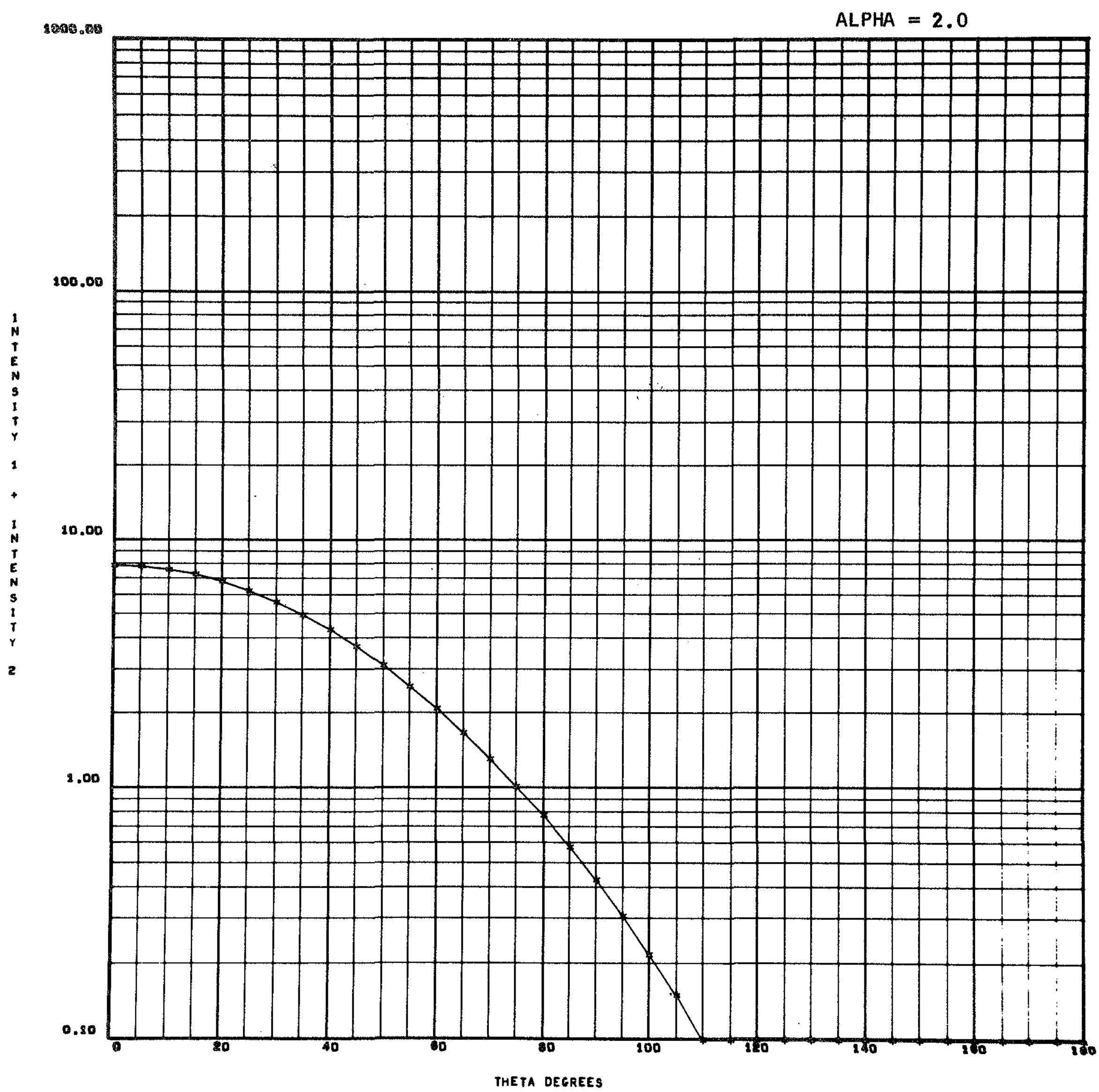

ALPHA = .9
1000.00
100.00
10.00
1.00
0.10
1000 TIMES INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

ALPHA = .9



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
POLARIZATION
0
20
40
60
80
100
120
140
160
180
200
THETA DEGREES


ALPHA = 2.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
THETA DEGREES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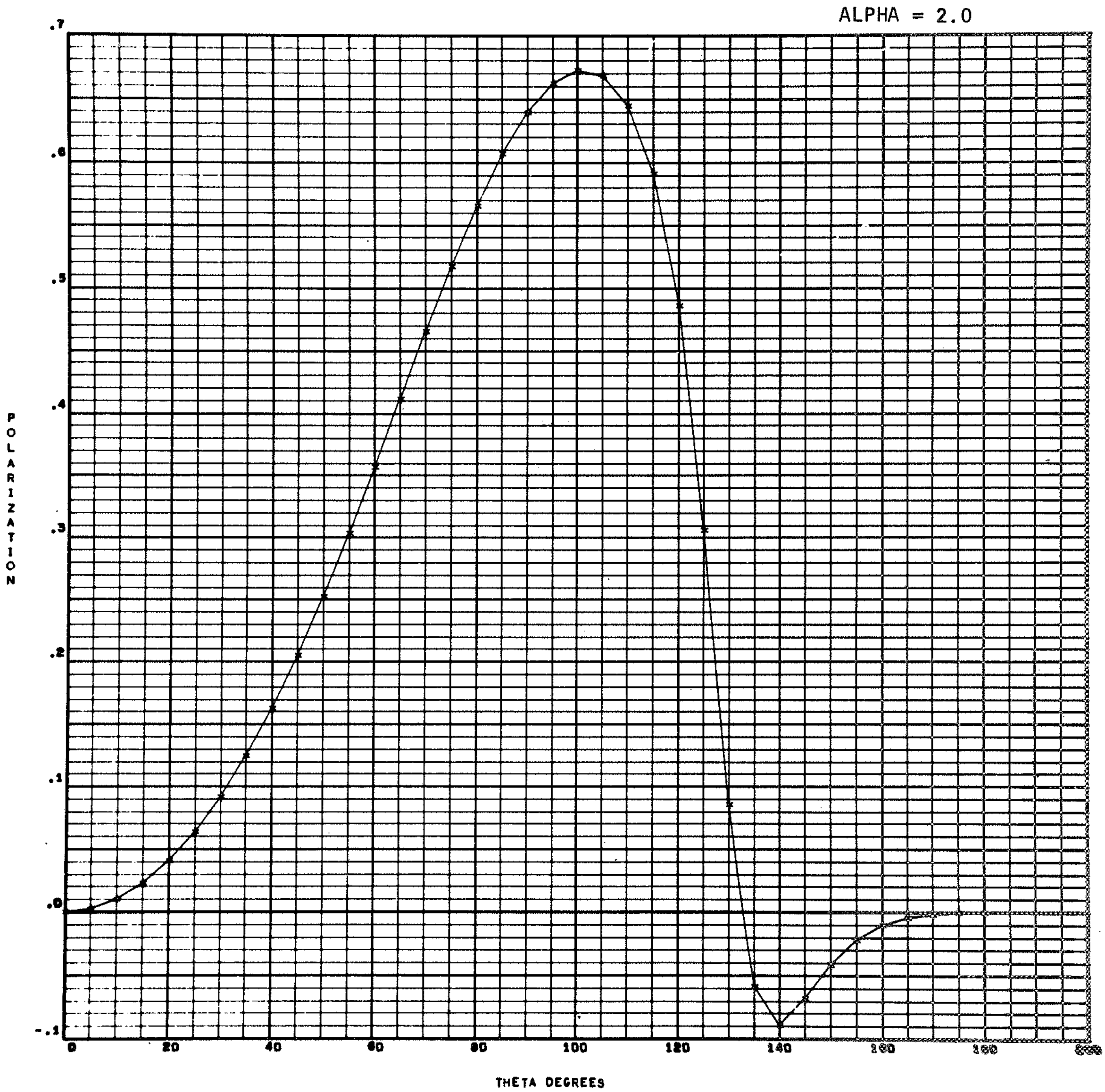
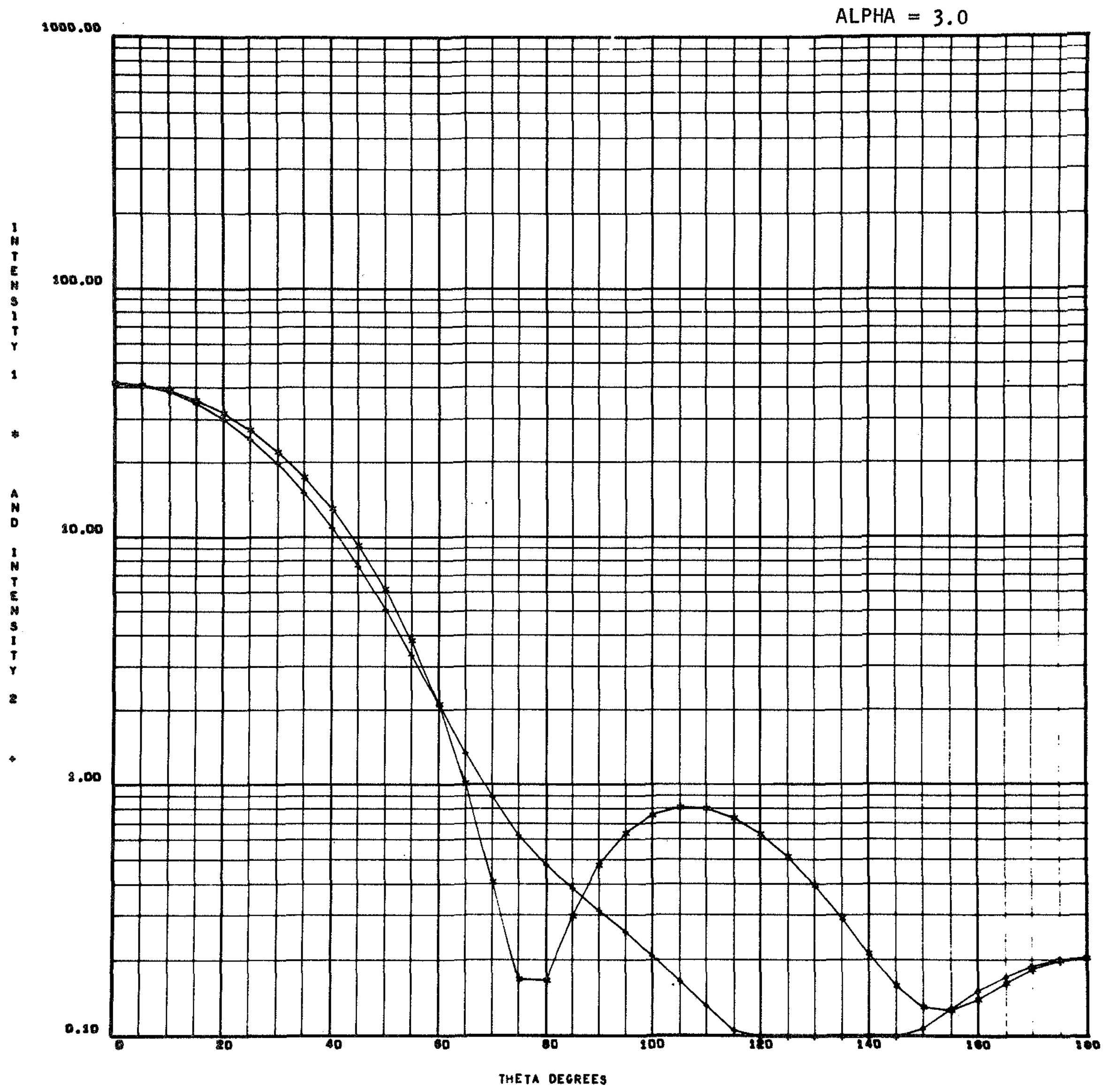
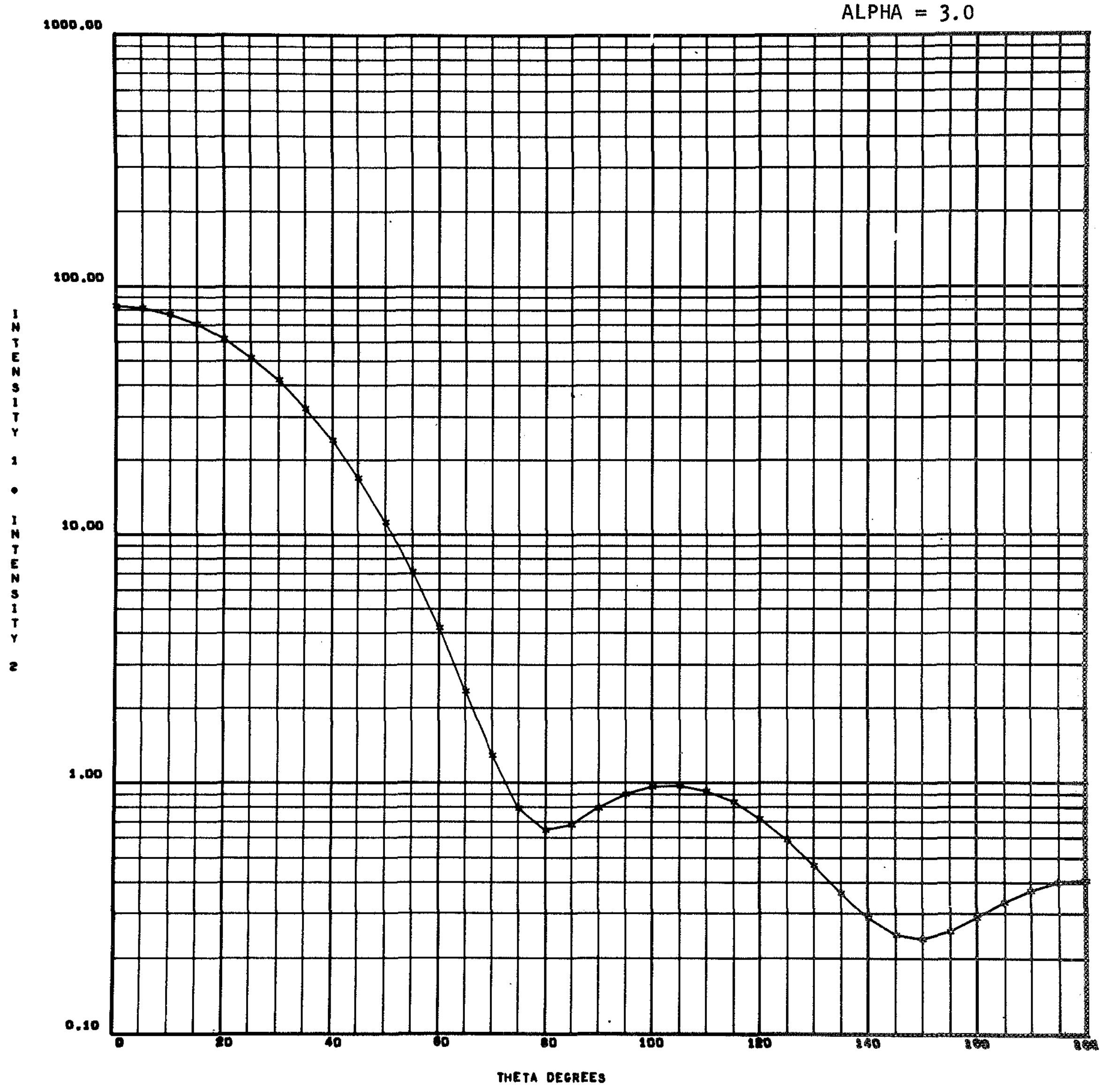
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

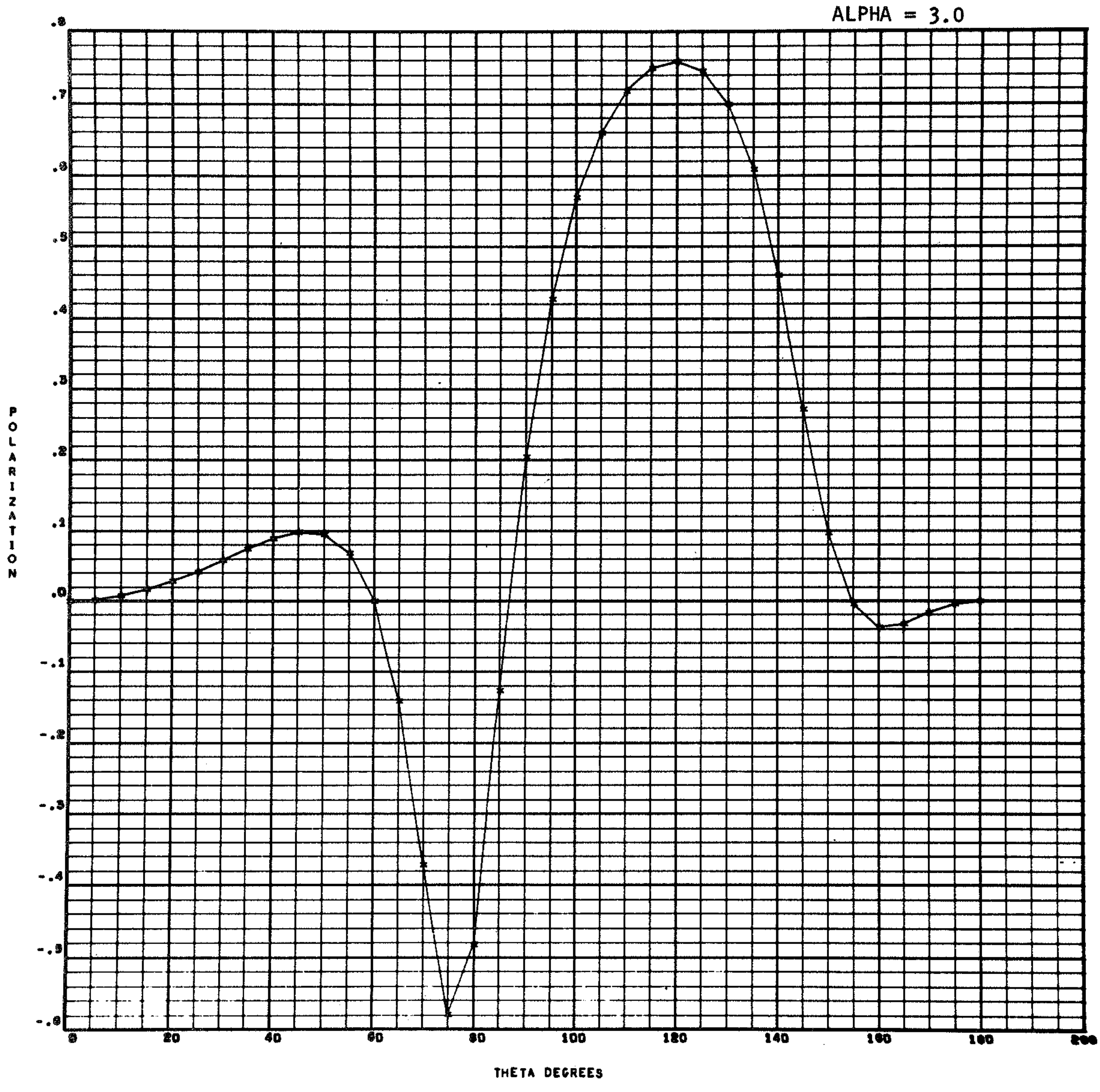

ALPHA = 2.0
POLARIZATION
THETA DEGREES
.7
.6
.5
.4
.3
.2
.1
.0
-.1
0
20
40
60
80
100
120
140


ALPHA = 3.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

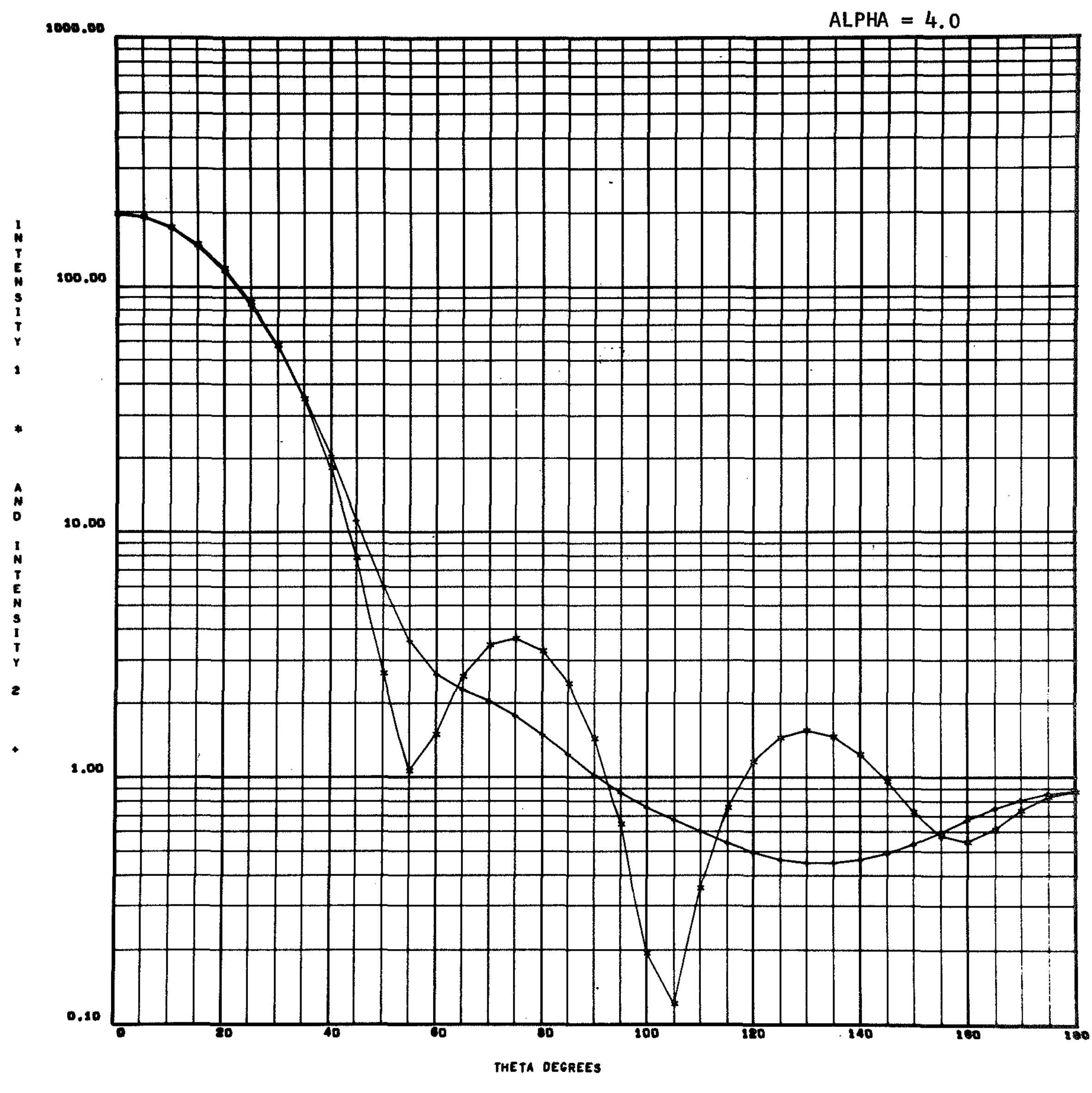
ALPHA = 3.0

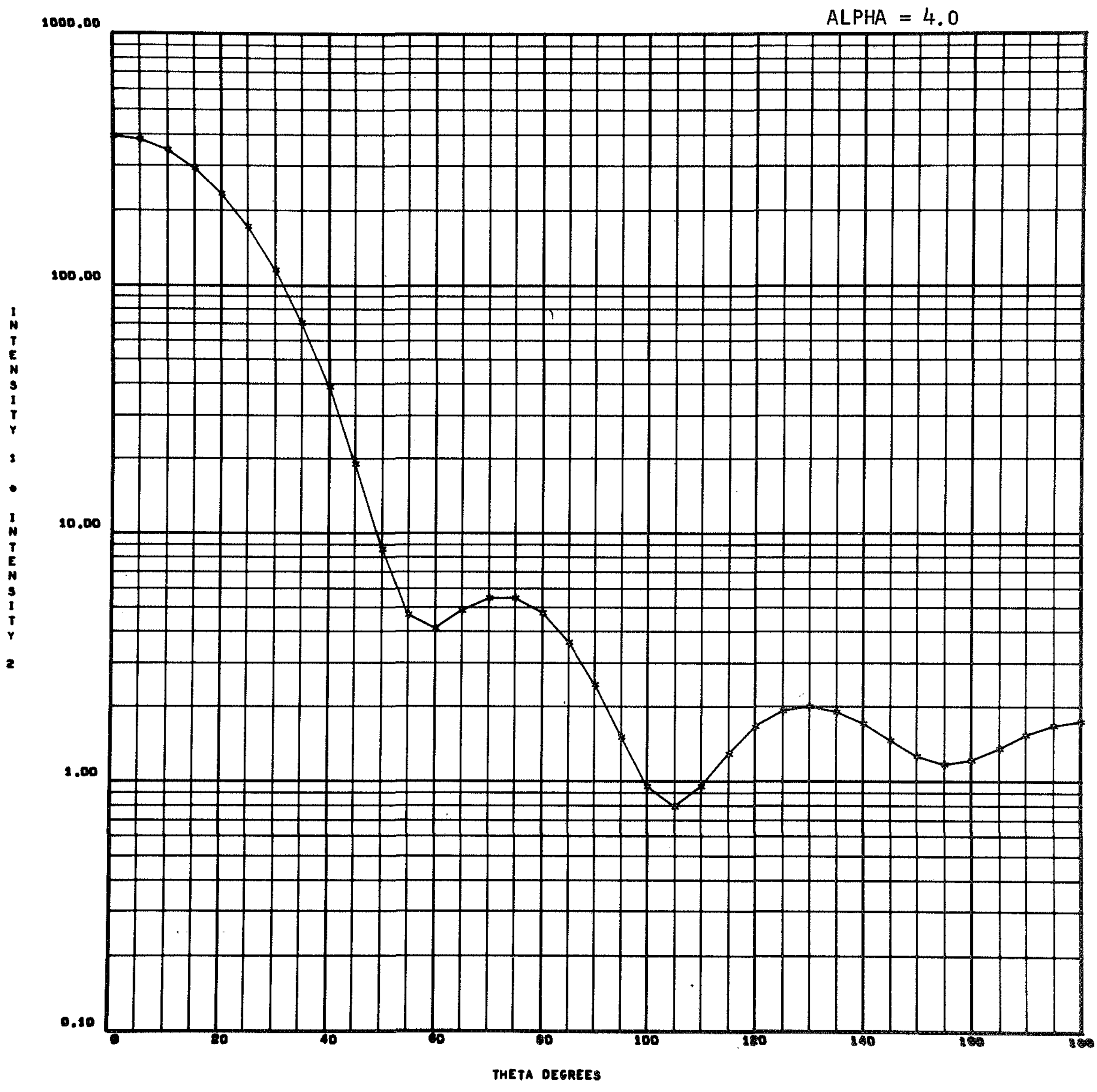


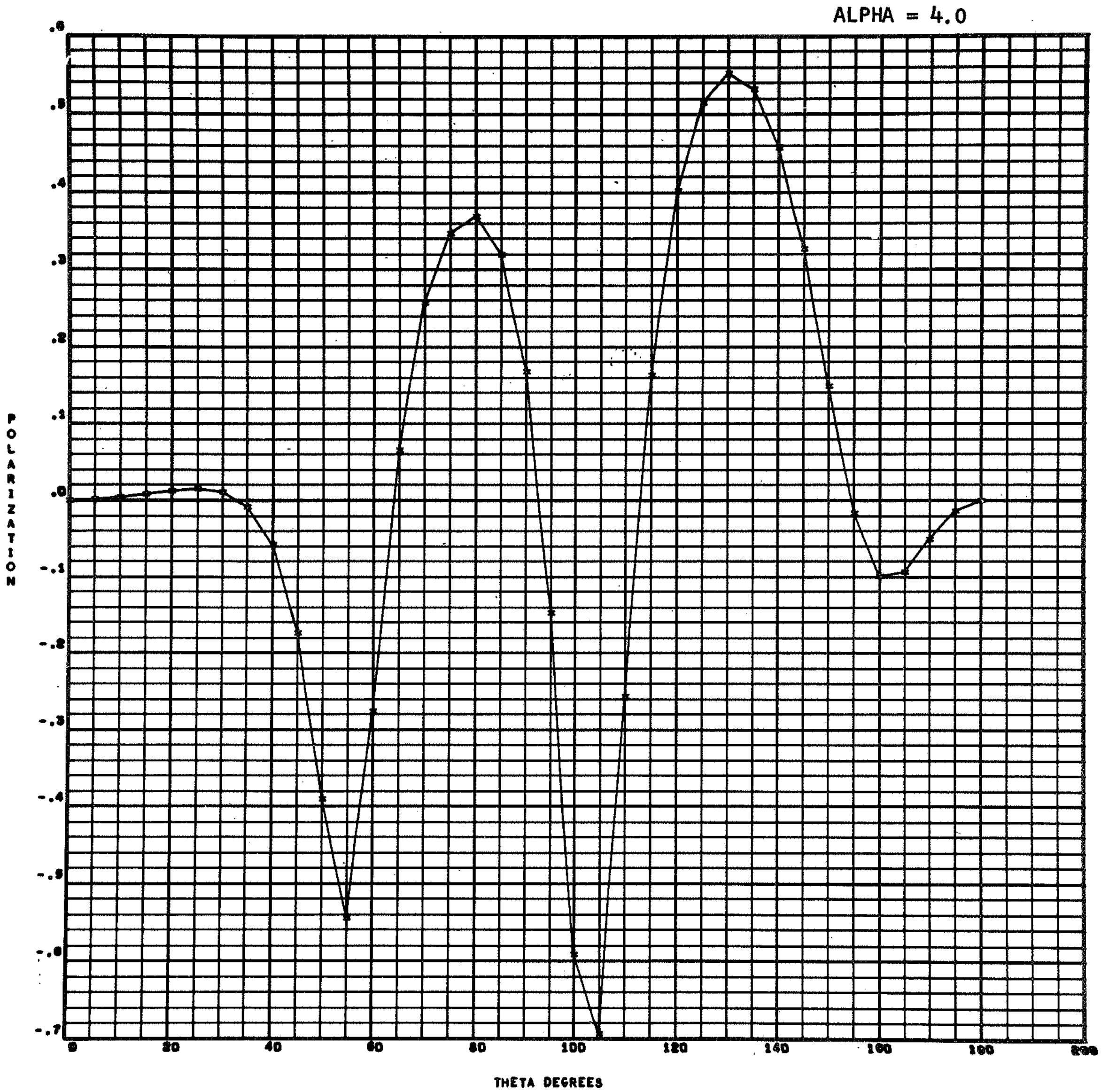
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

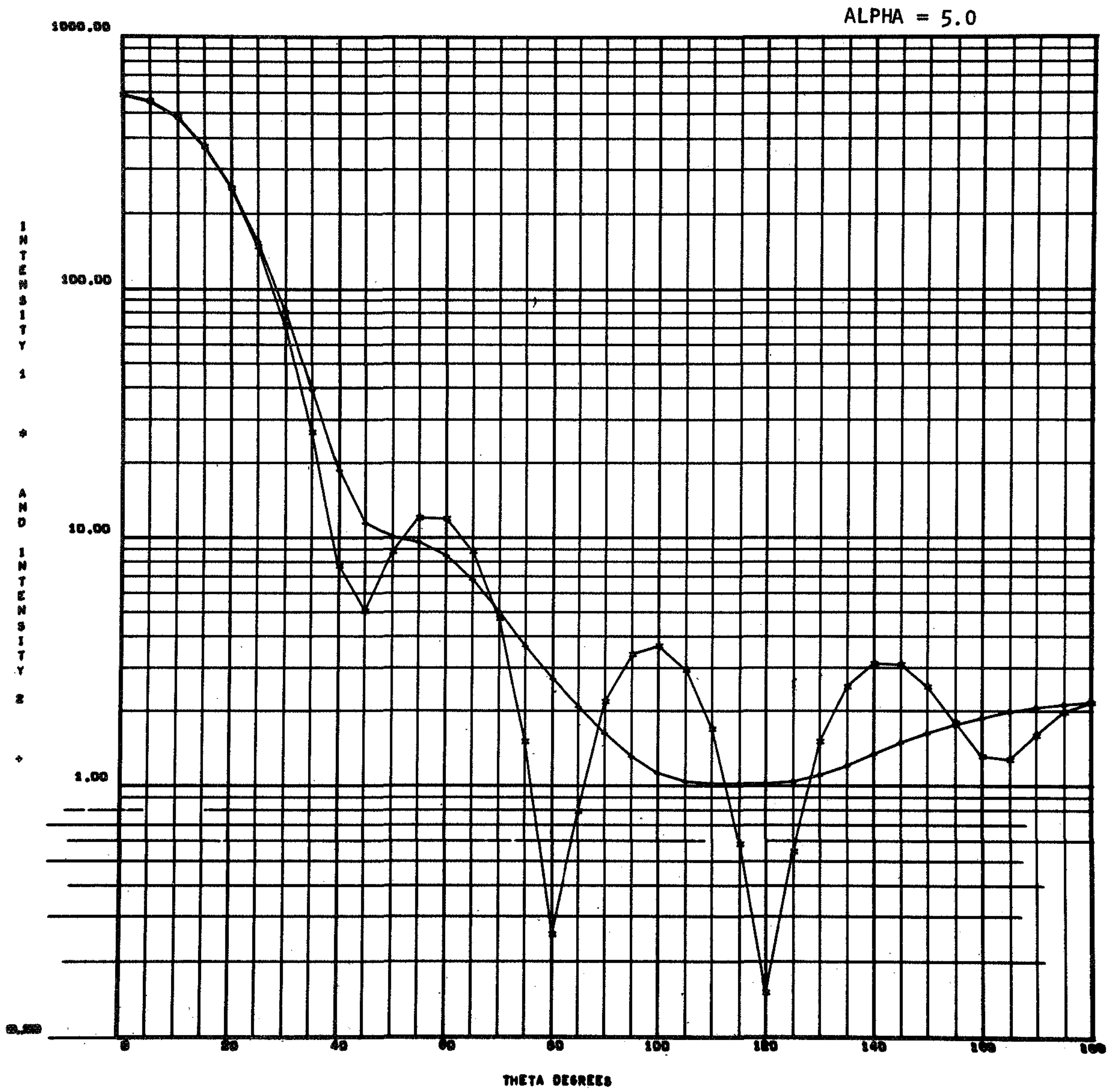


ALPHA = 3.0
POLARIZATION
.8
.7
.6
.5
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
-.6
0
20
40
60
80
100
120
140
160
180
200
THETA DEGREES

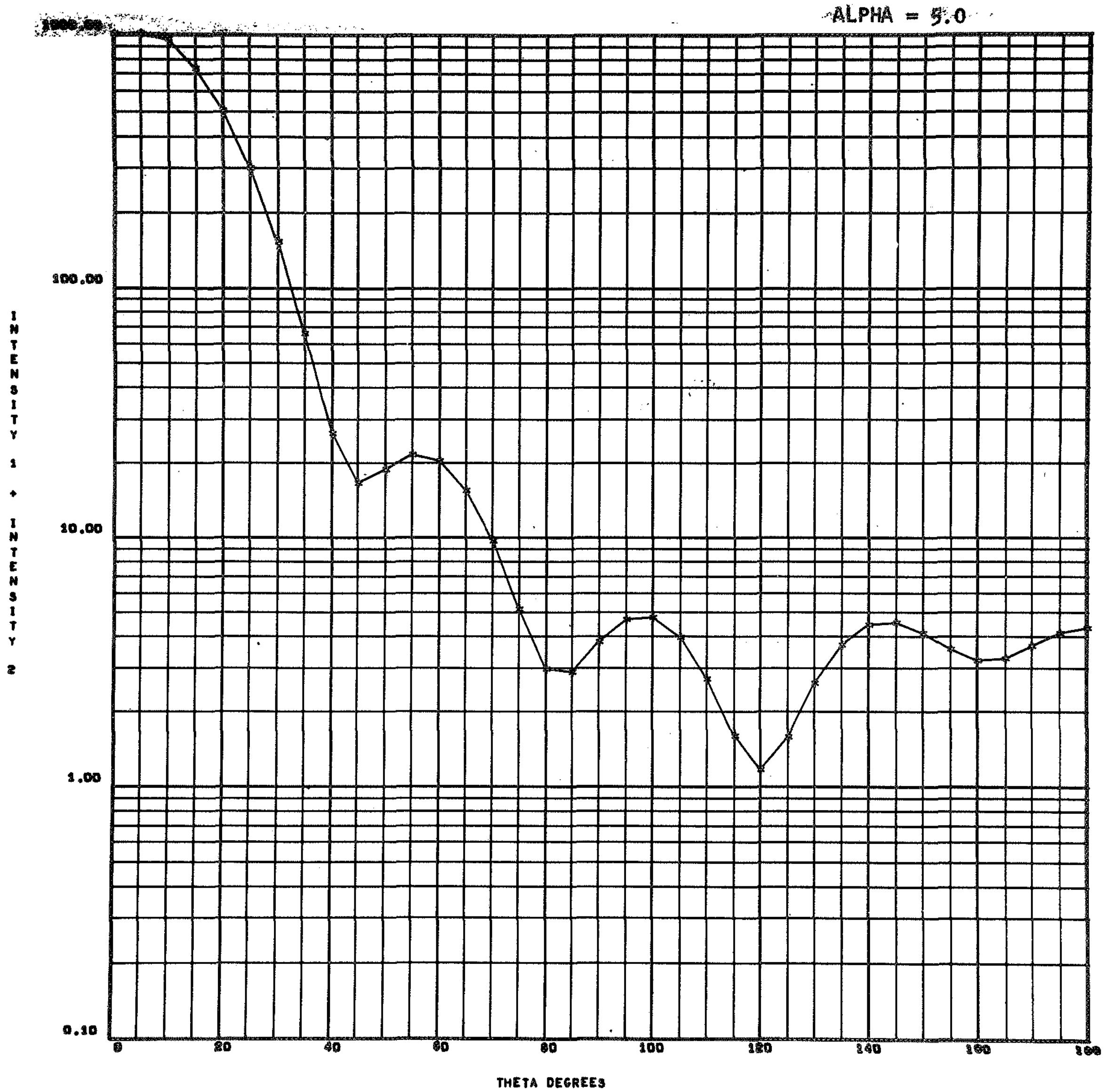

ALPHA = 4.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
THETA DEGREES


ALPHA = 4.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

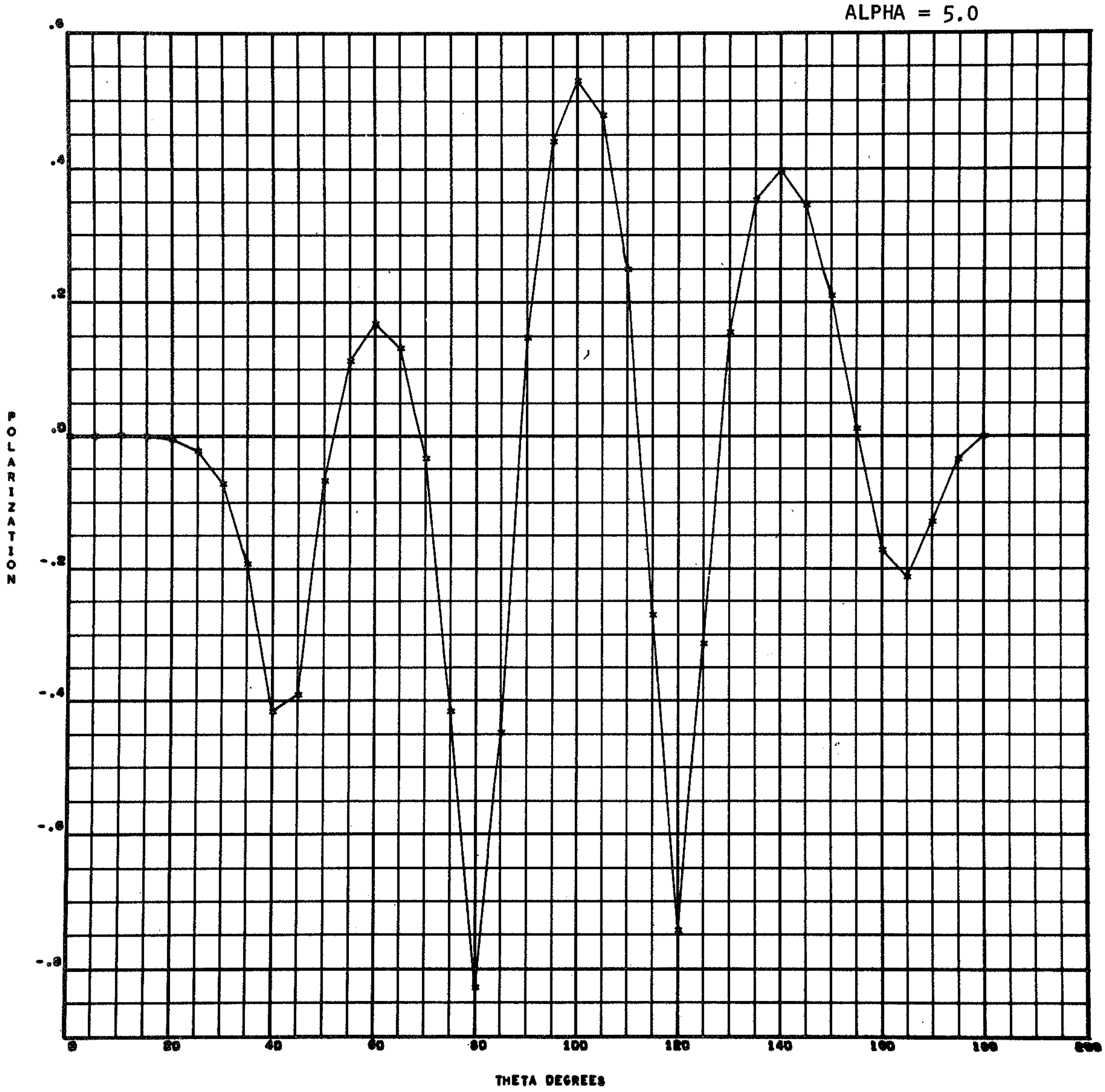

ALPHA = 4.0
POLARIZATION
THETA DEGREES
.6
.5
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
0
20
40
60
80
100
120
140
160
180
200



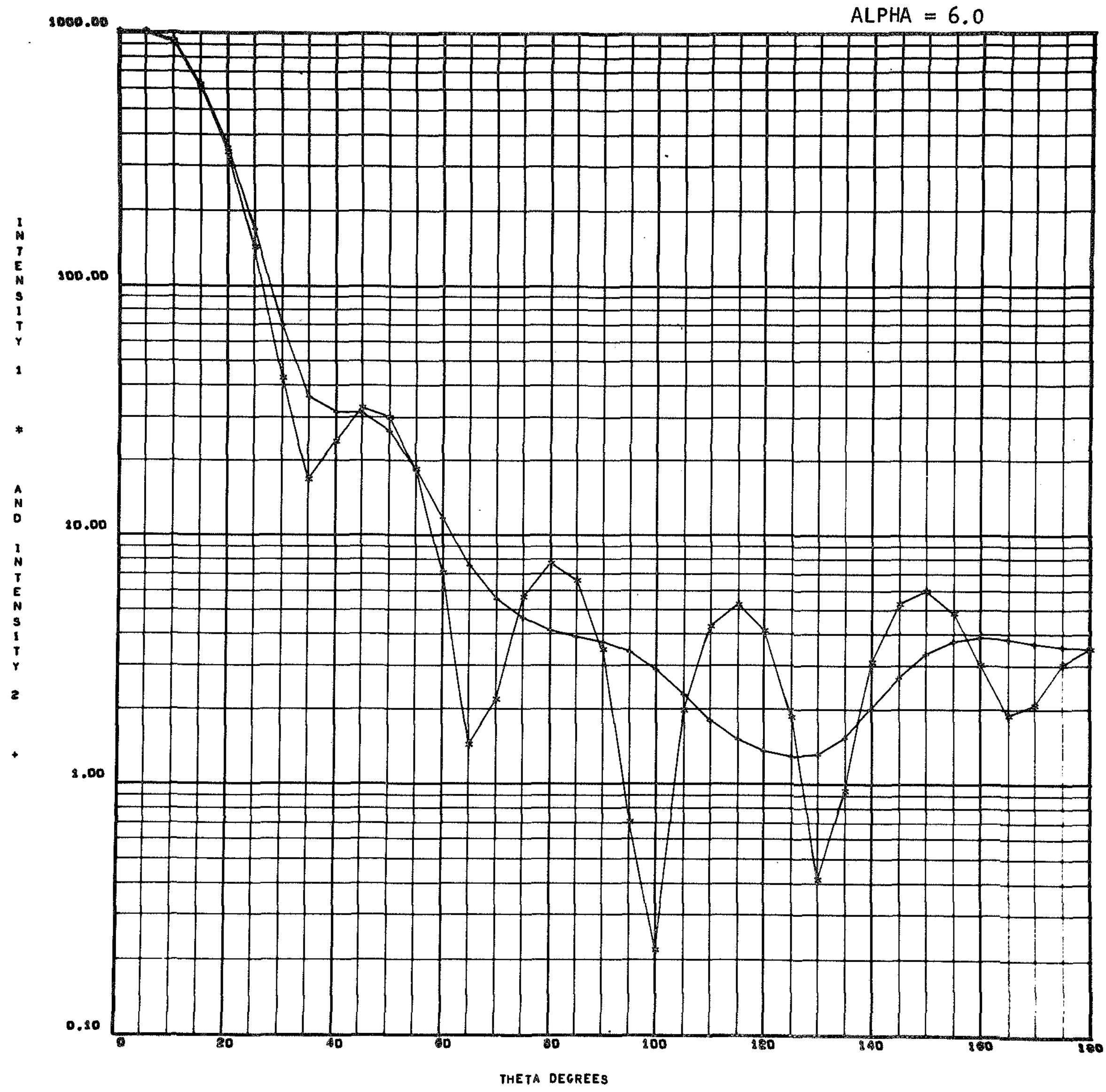
ALPHA = 5.0
1000.00
100.00
10.00
1.00
INTENSITY 1 + AND INTENSITY 2 +
THETA DEGREES

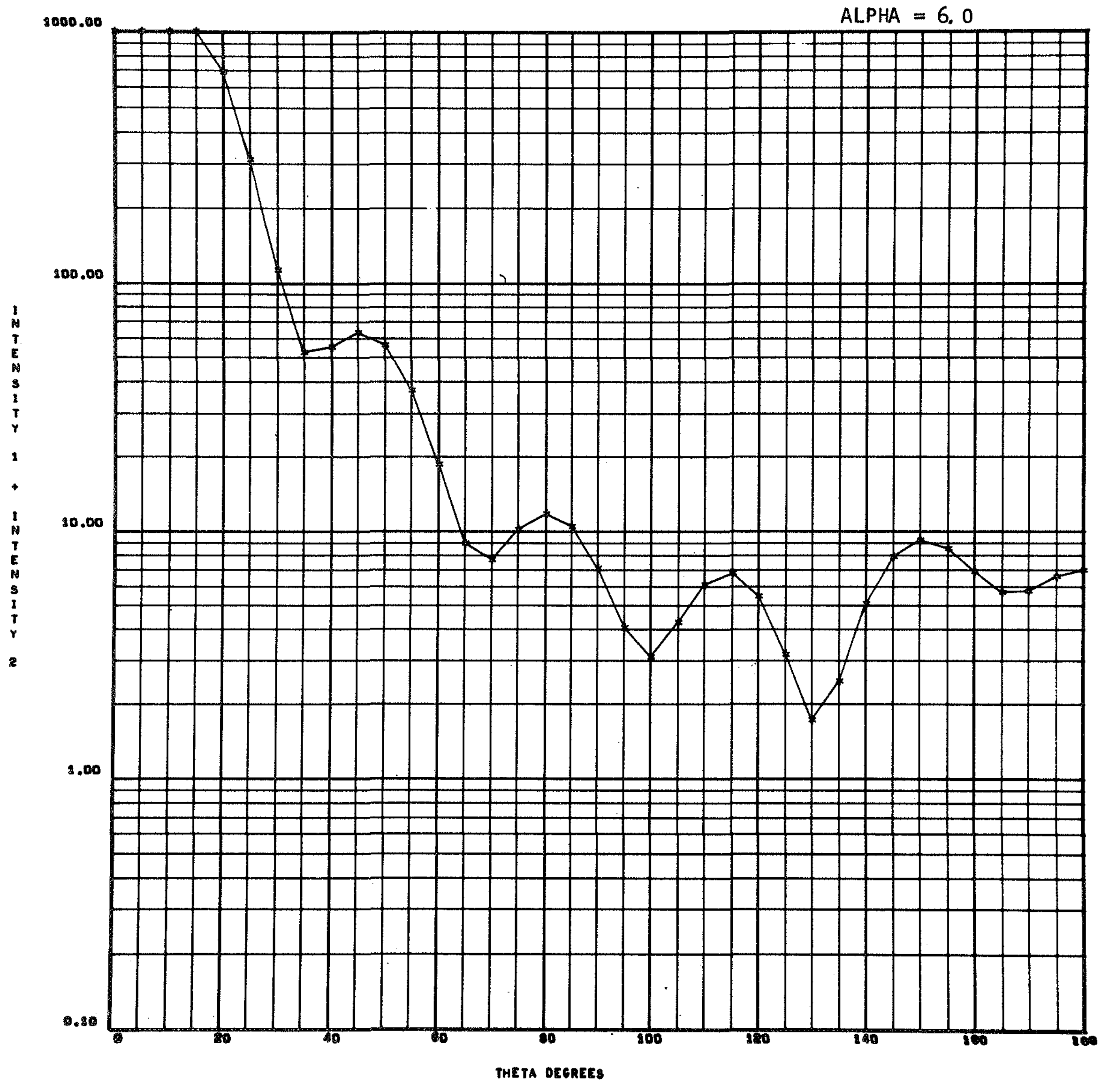

ALPHA = 5.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

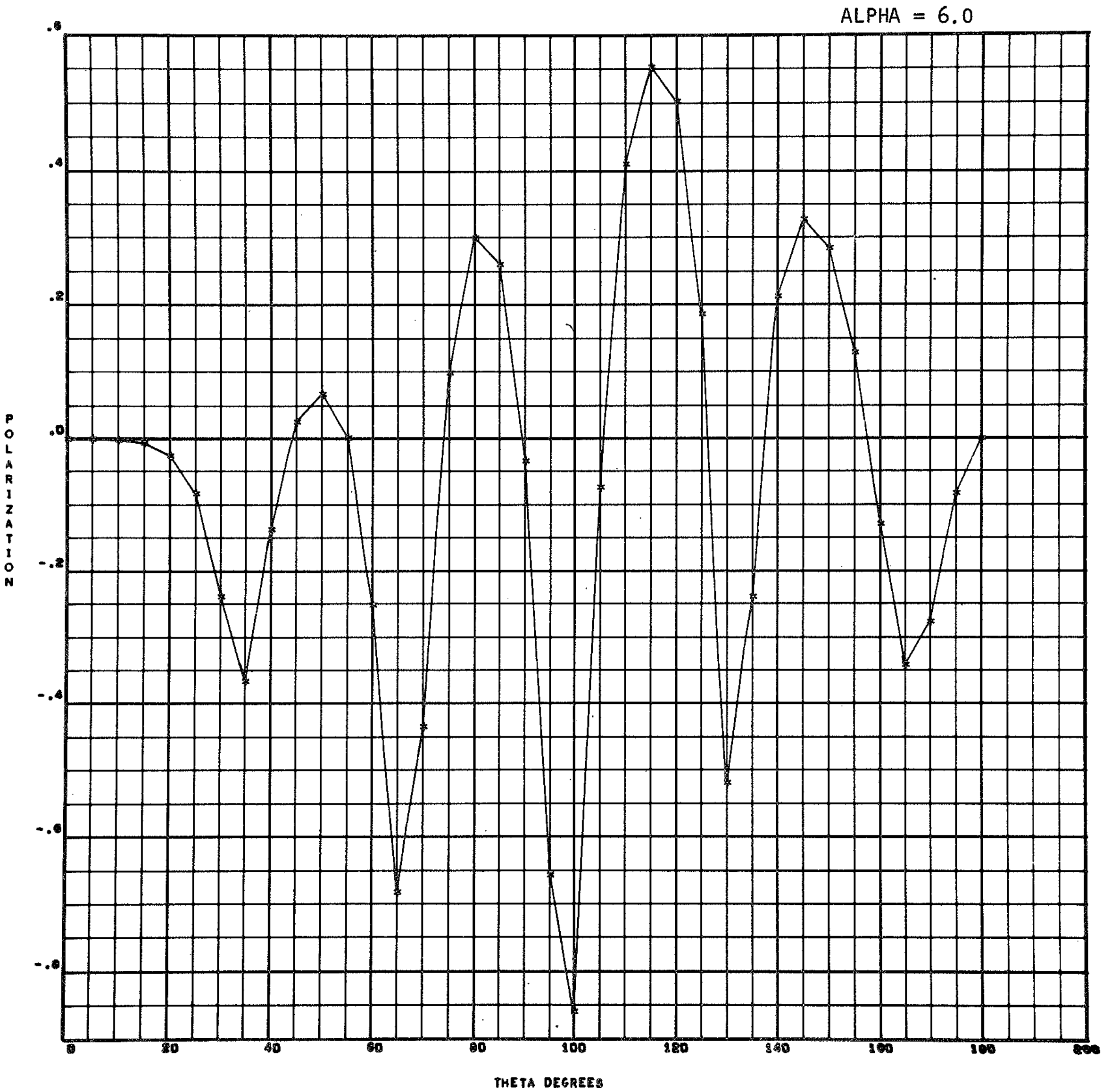
ALPHA = 5.0

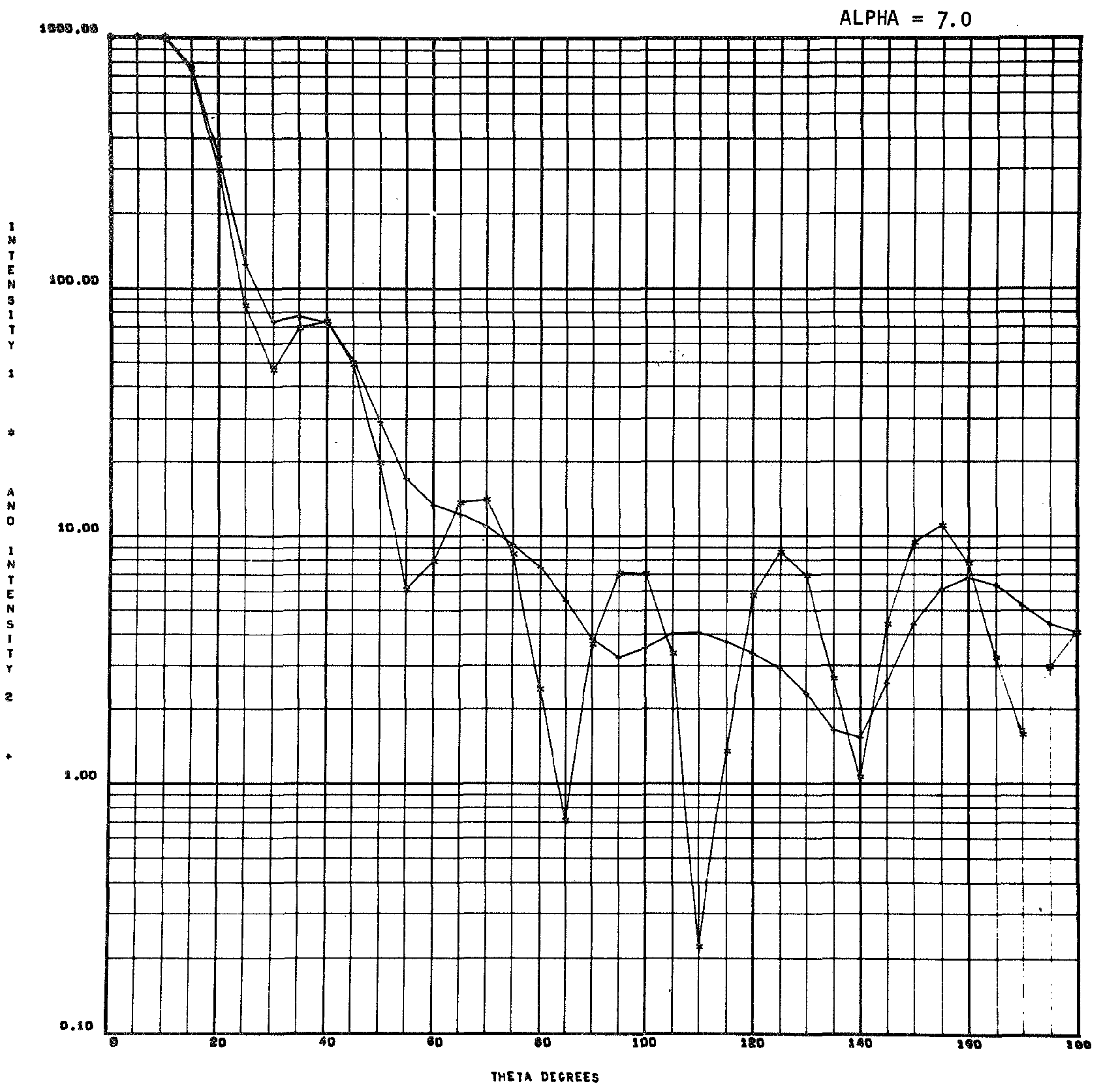


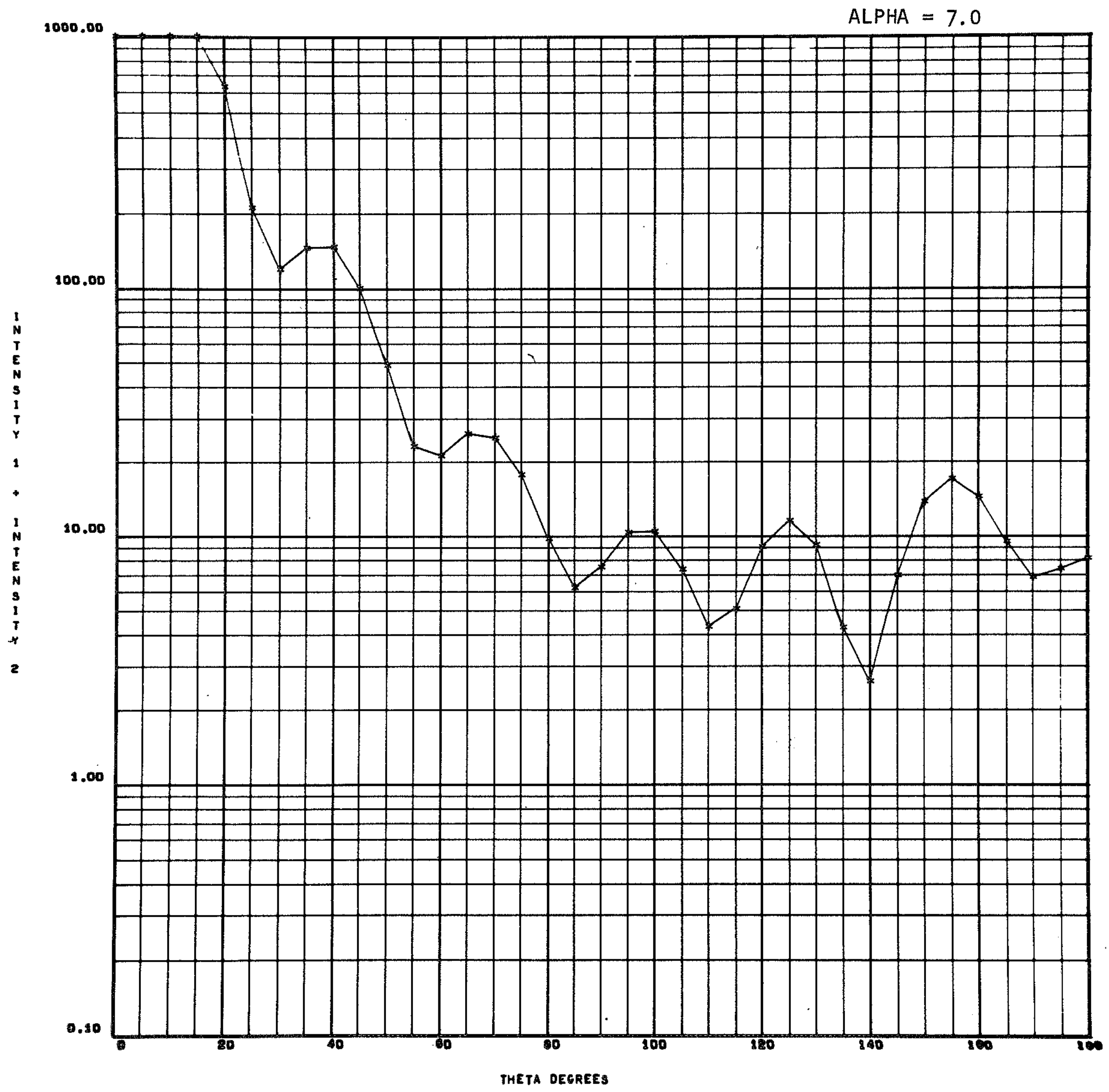
.6
.4
.2
.0
-.2
-.4
-.6
-.8
POLARIZATION
0
20
40
60
80
100
120
140
160
180
200
THETA DEGREES

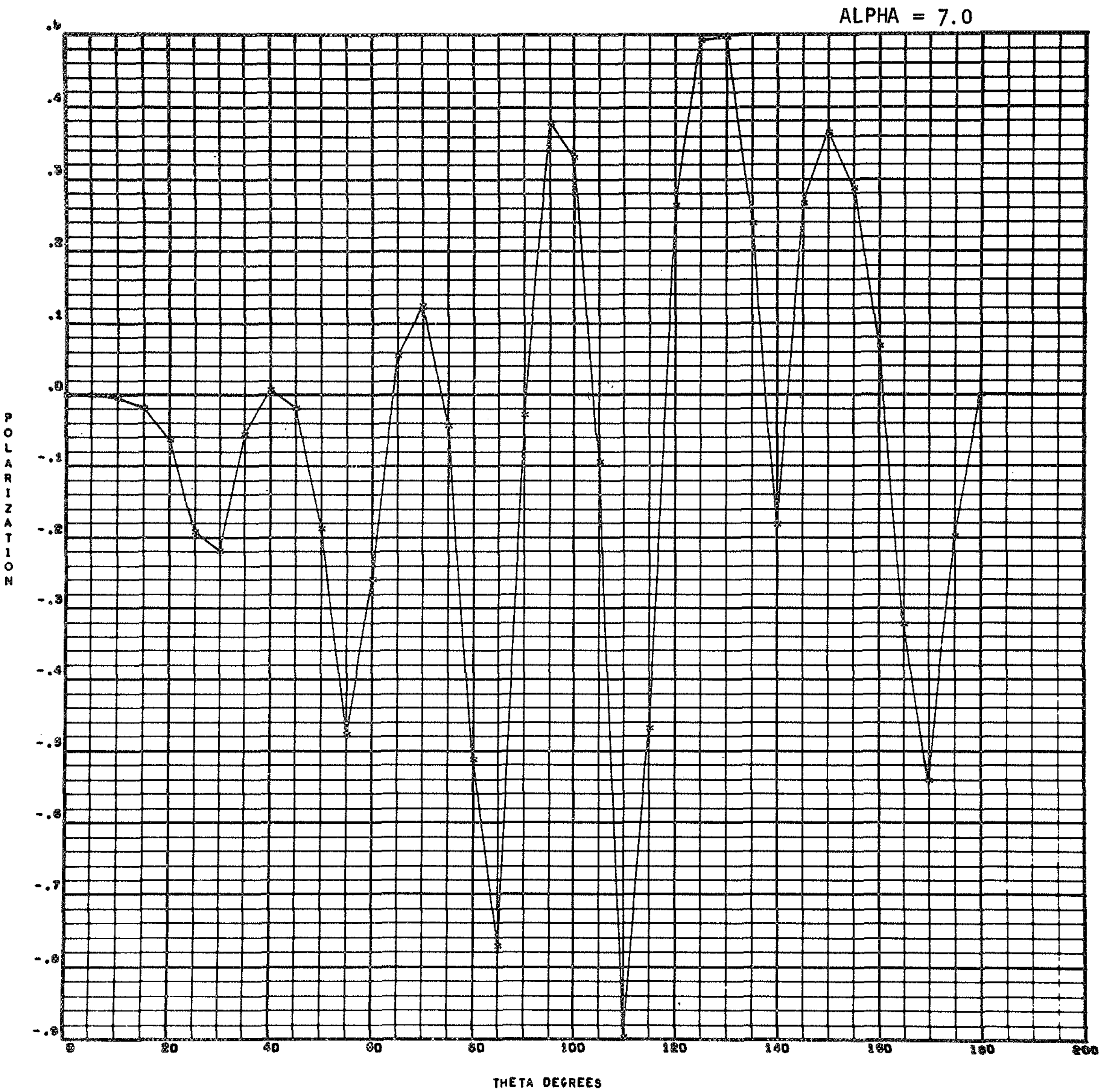

ALPHA = 6.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

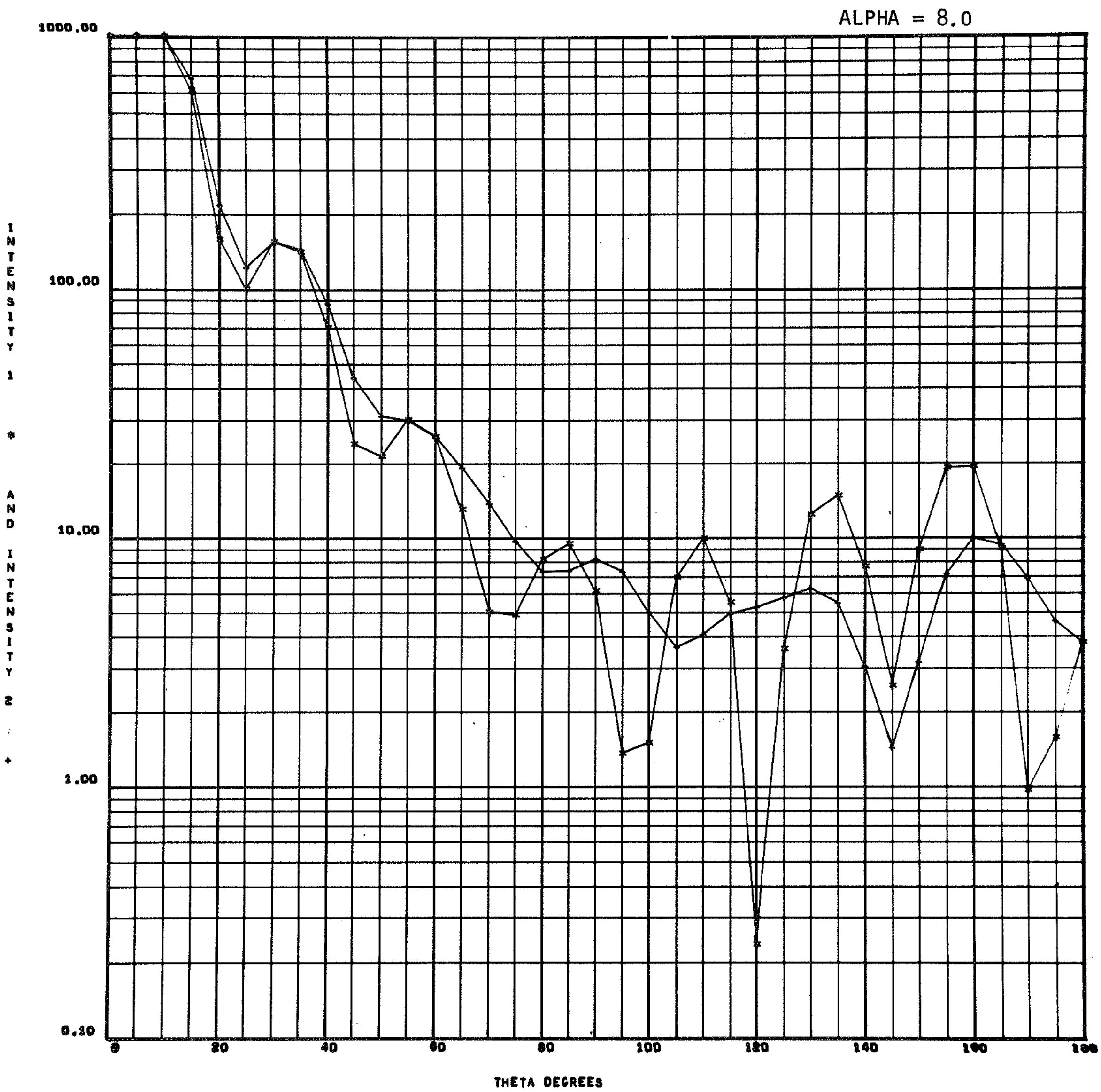

ALPHA = 6.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

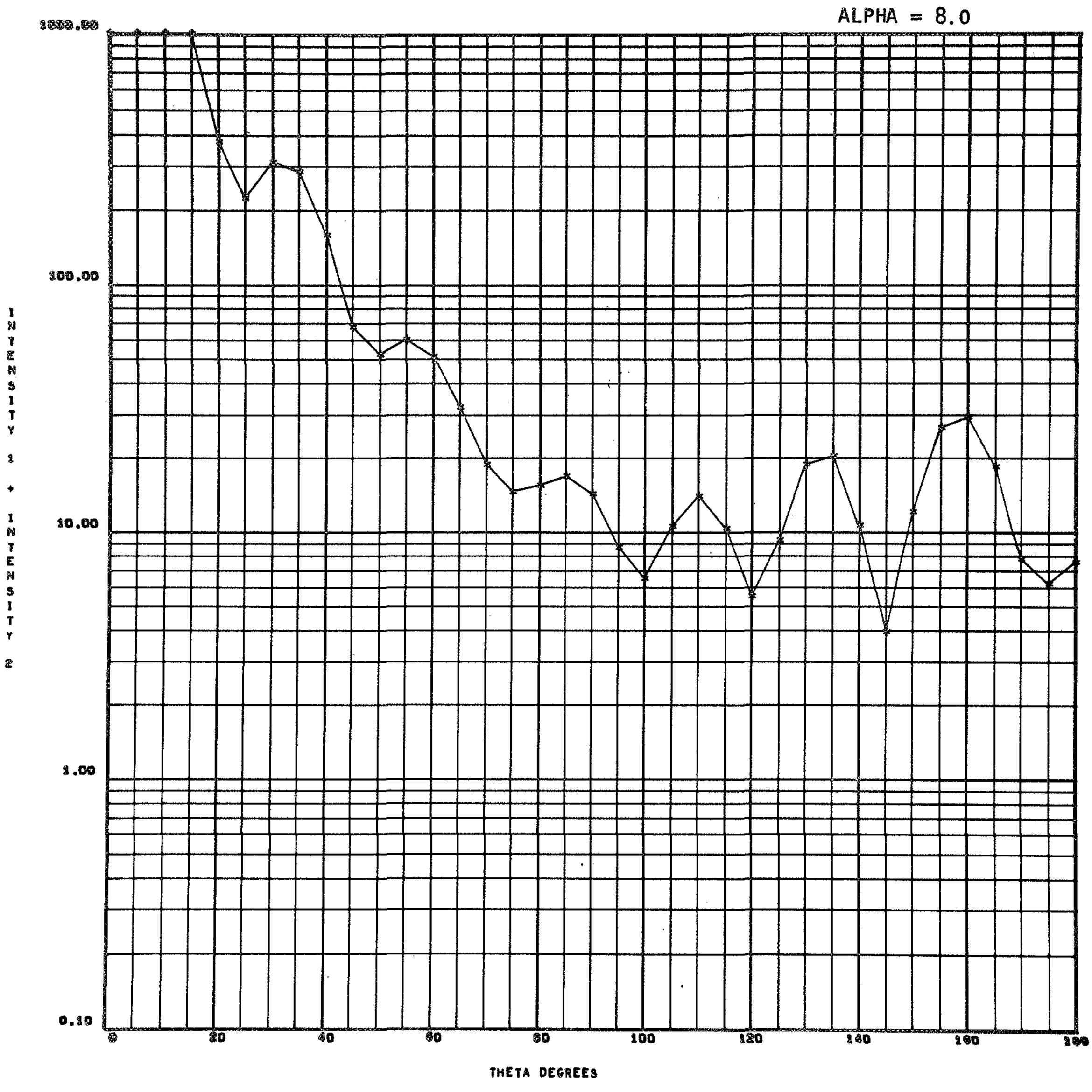

ALPHA = 6.0
POLARIZATION
THETA DEGREES
.6
.4
.2
.0
-.2
-.4
-.6
-.8
0
20
40
60
80
100
120
140
160
180
200

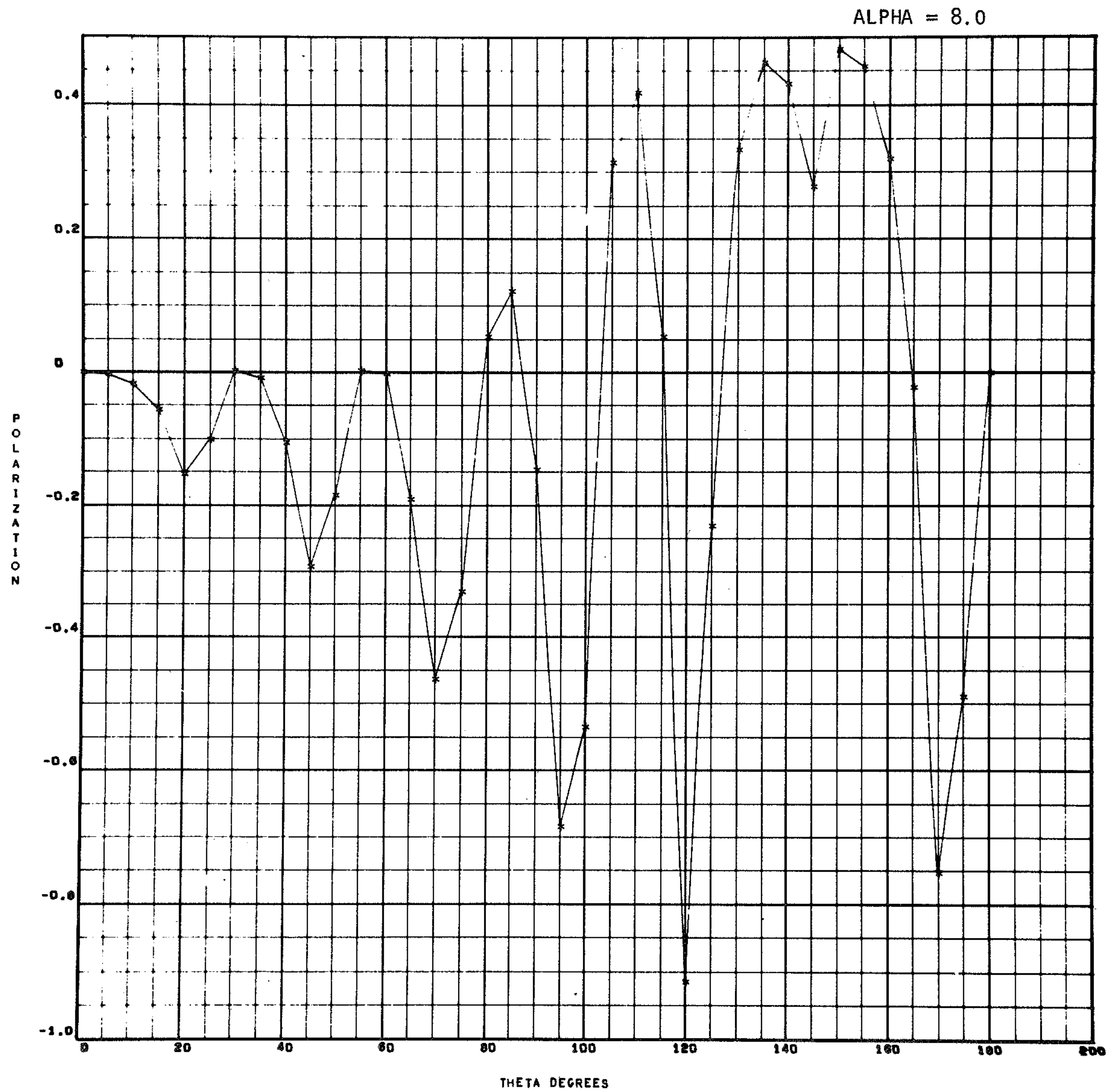

ALPHA = 7.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
THETA DEGREES


ALPHA = 7.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES


ALPHA = 7.0
POLARIZATION
THETA DEGREES
.5
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
-.8
-.9
0
20
40
60
80
100
120
140
160
180
200


ALPHA = 8.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
THETA DEGREES


ALPHA = 8.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES


ALPHA = 8.0
POLARIZATION
THETA DEGREES
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
0
20
40
60
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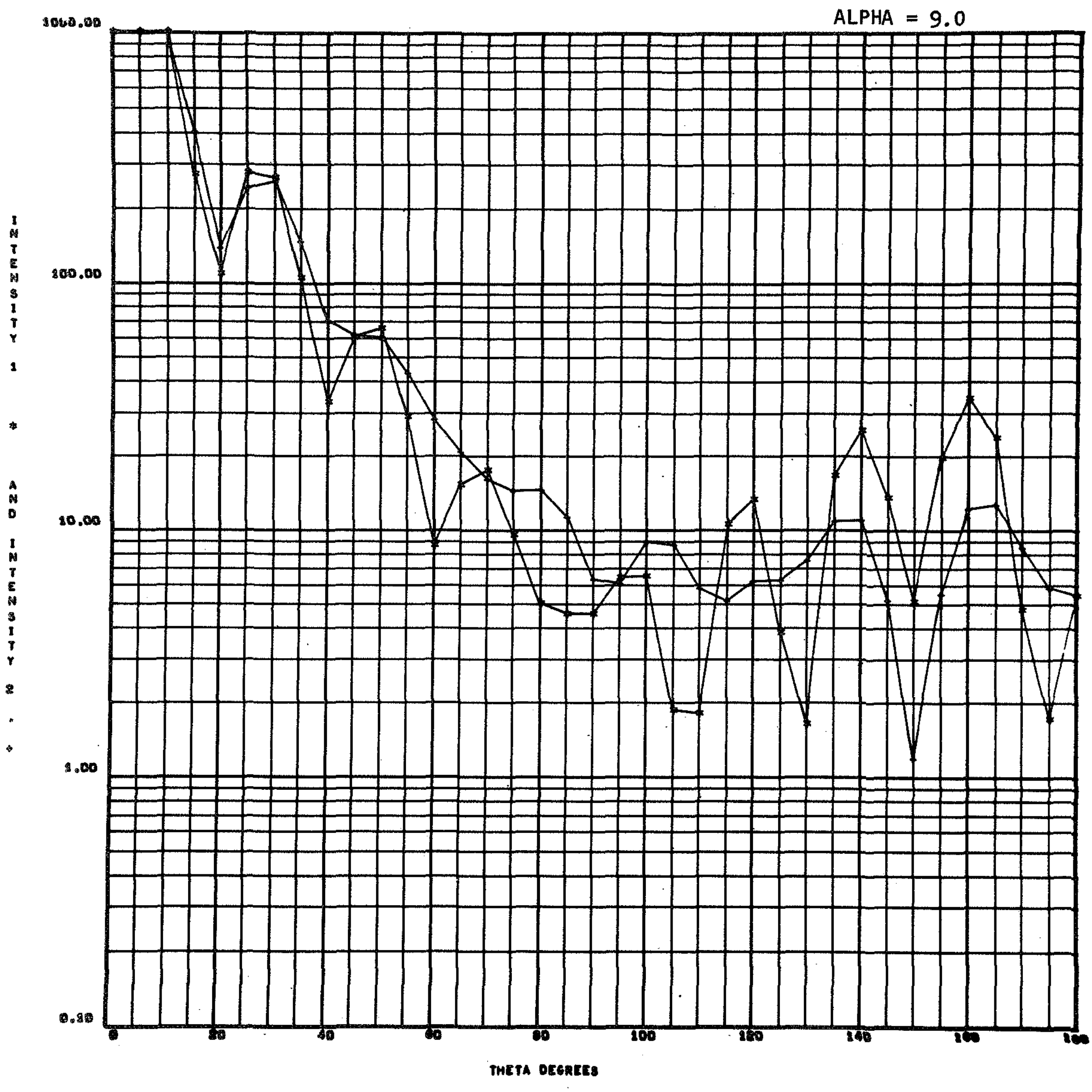
100
120
140
160
180
200


ALPHA = 9.0
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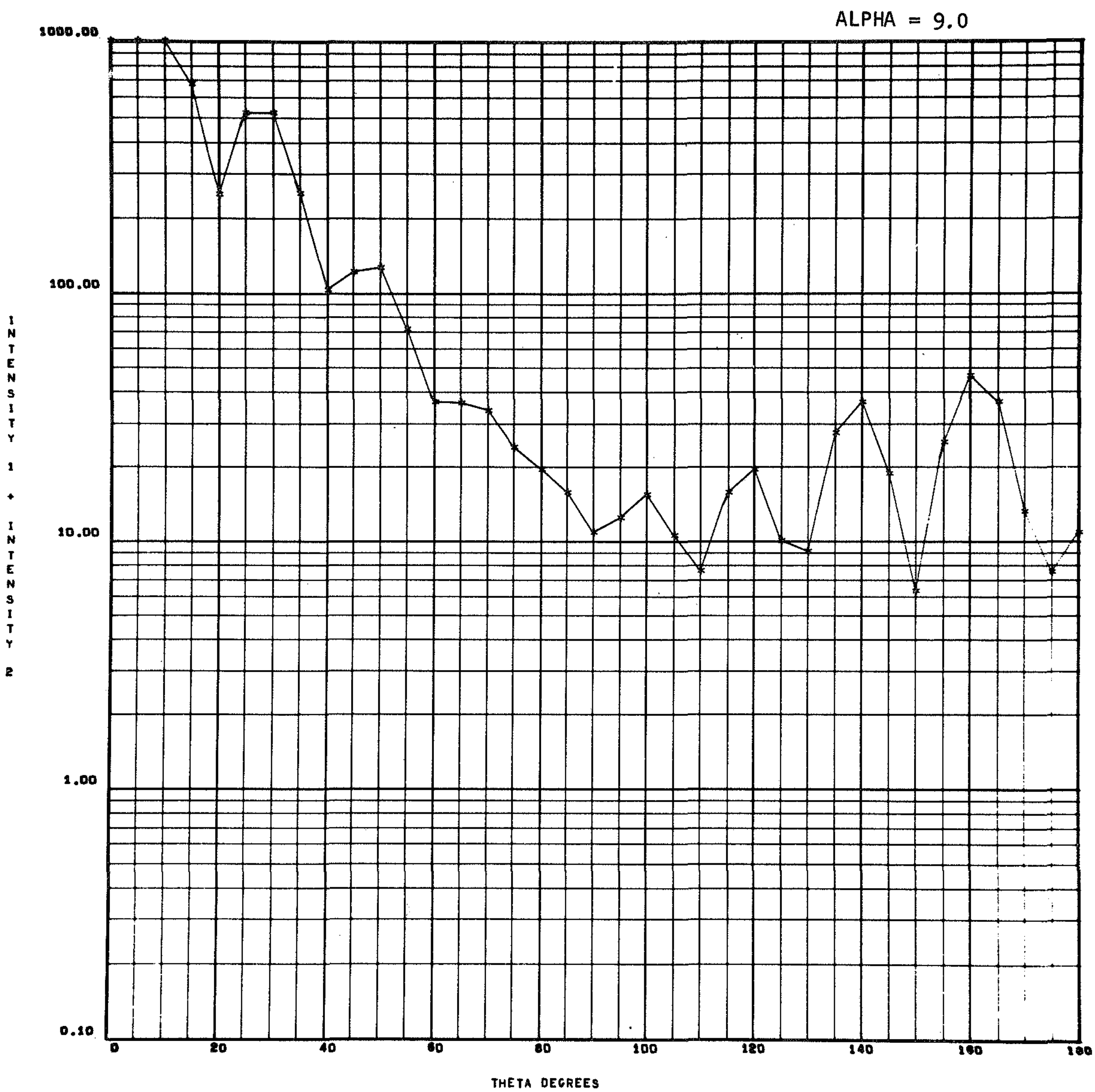
100.00
10.00
1.00
0.10
INTENSITY 1 * AND INTENSITY 2 , +
0
20
40
60
80
100
120
140
THETA DEGREES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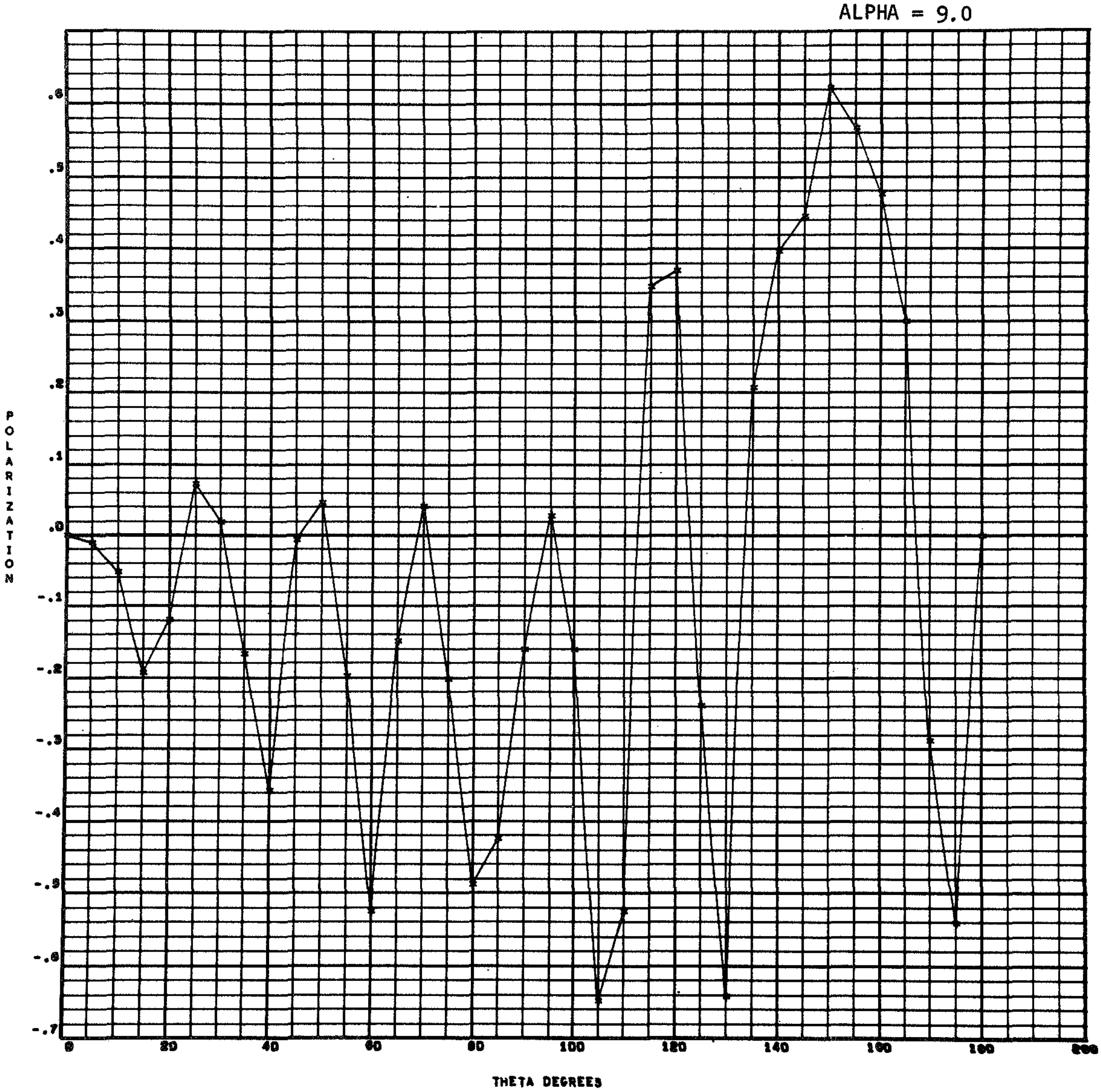
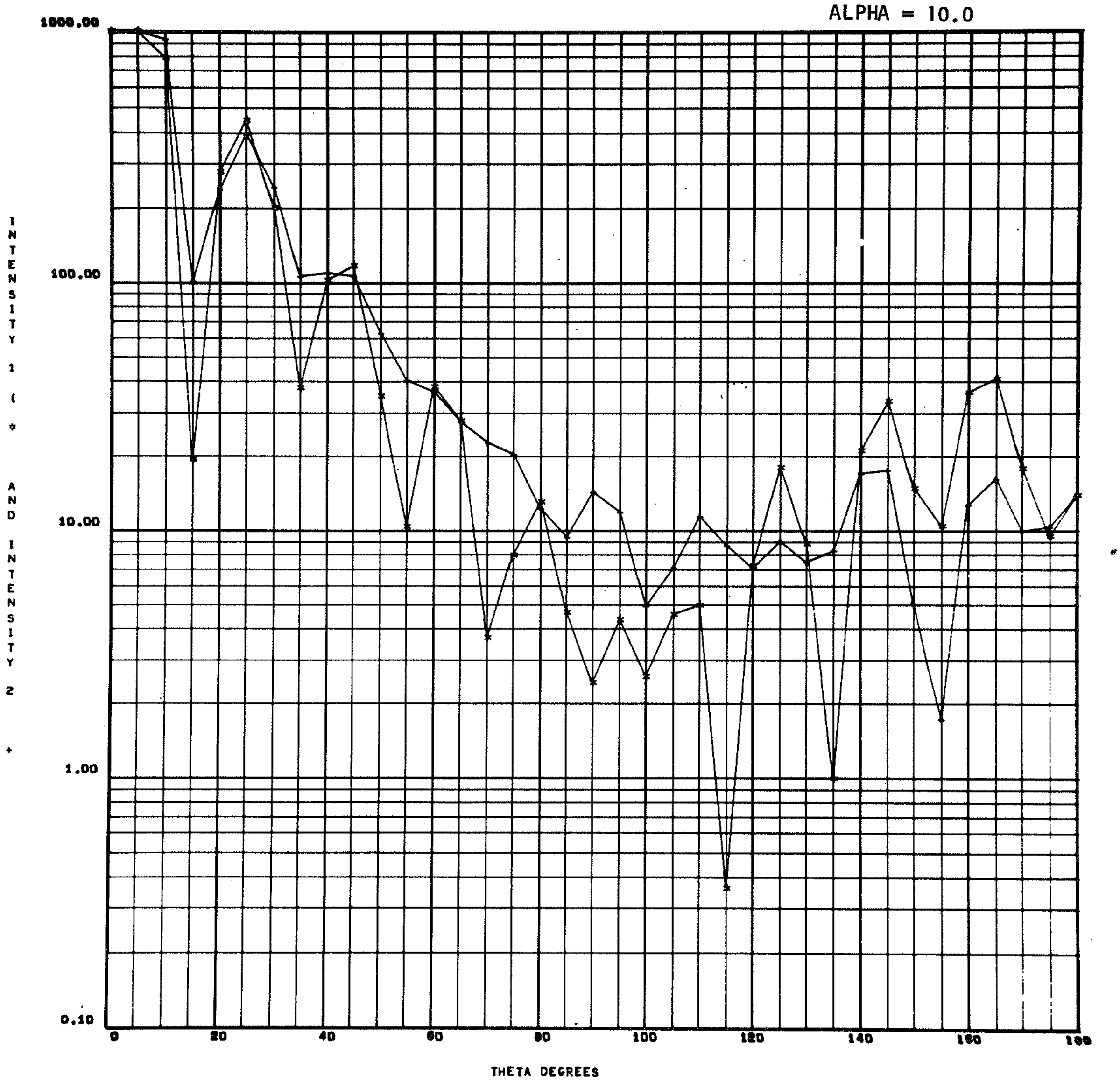
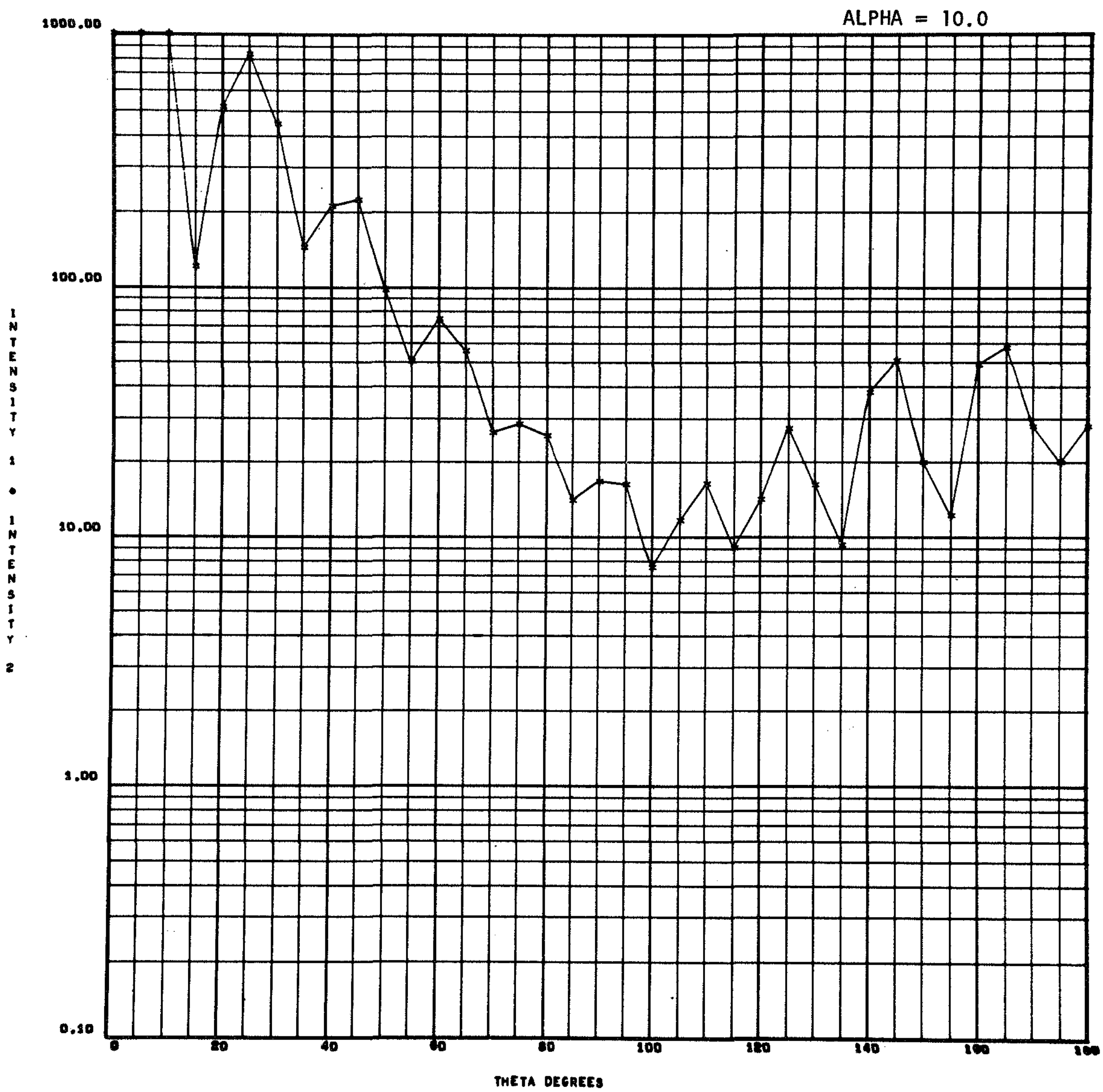
100.00
10.00
1.00
0.10
INTENSITY 1 + INTENSITY 2
0
20
40
60
80
100
120
140
160
180
THETA DEGREES

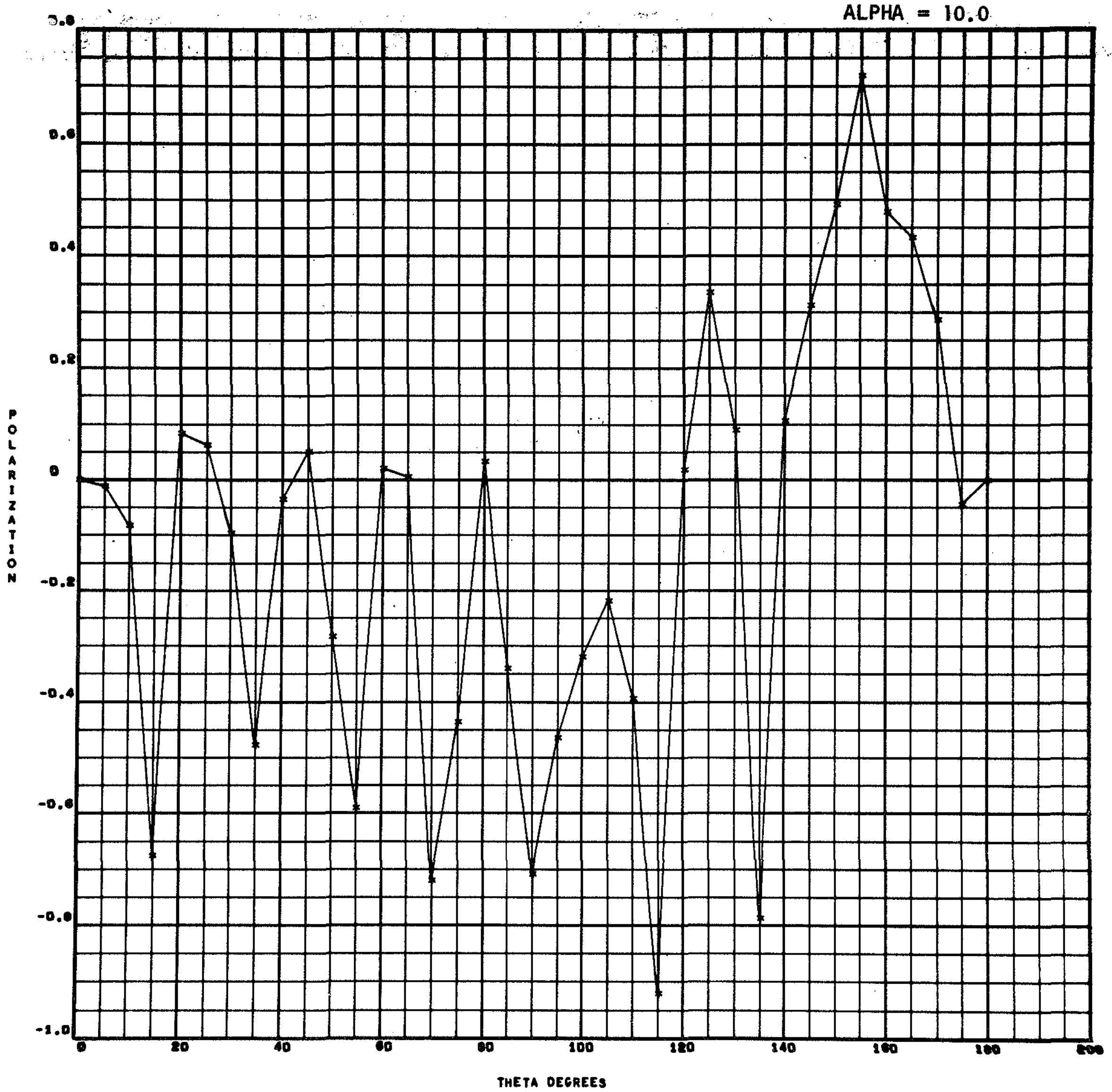
ALPHA = 9.0

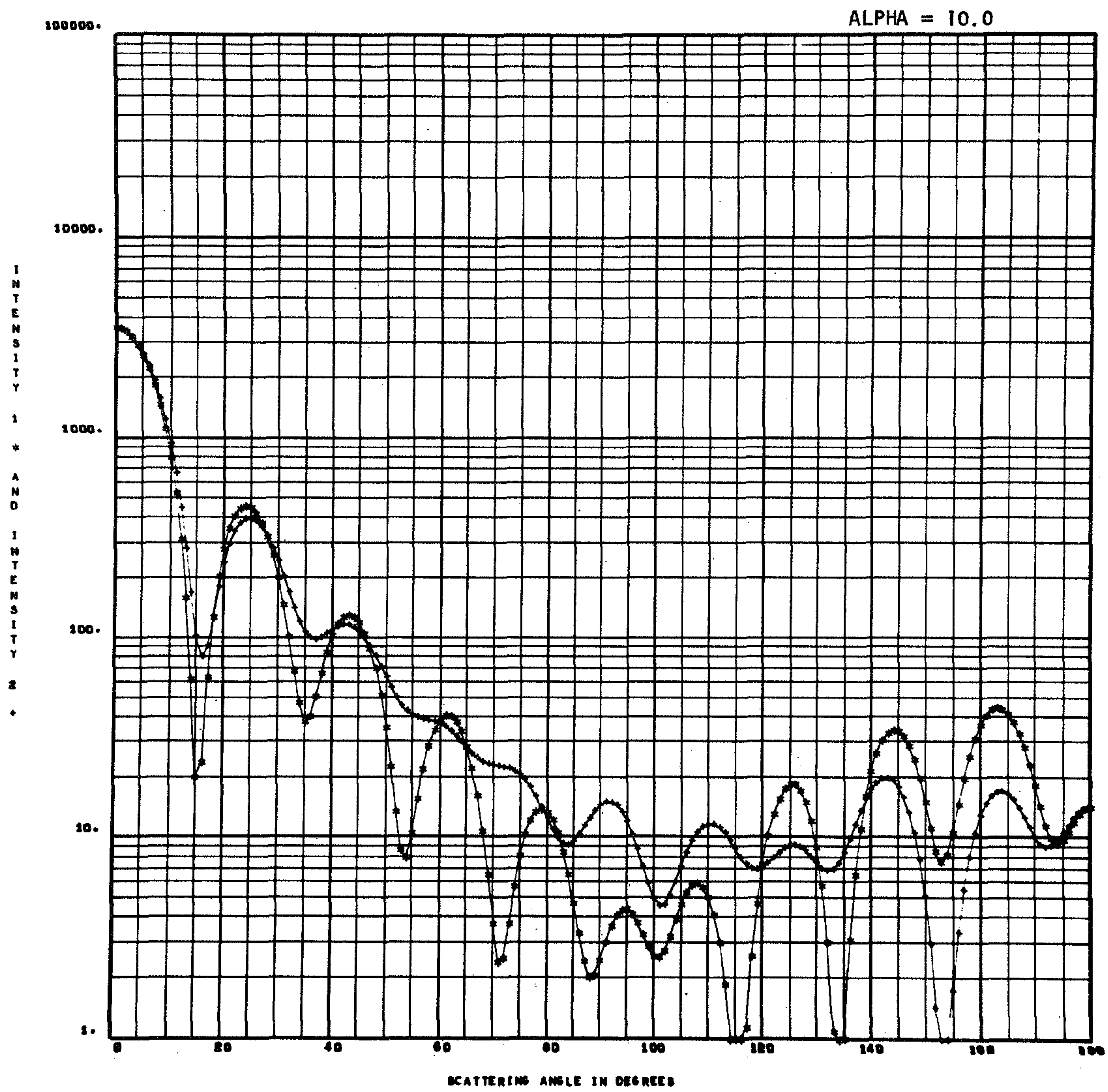


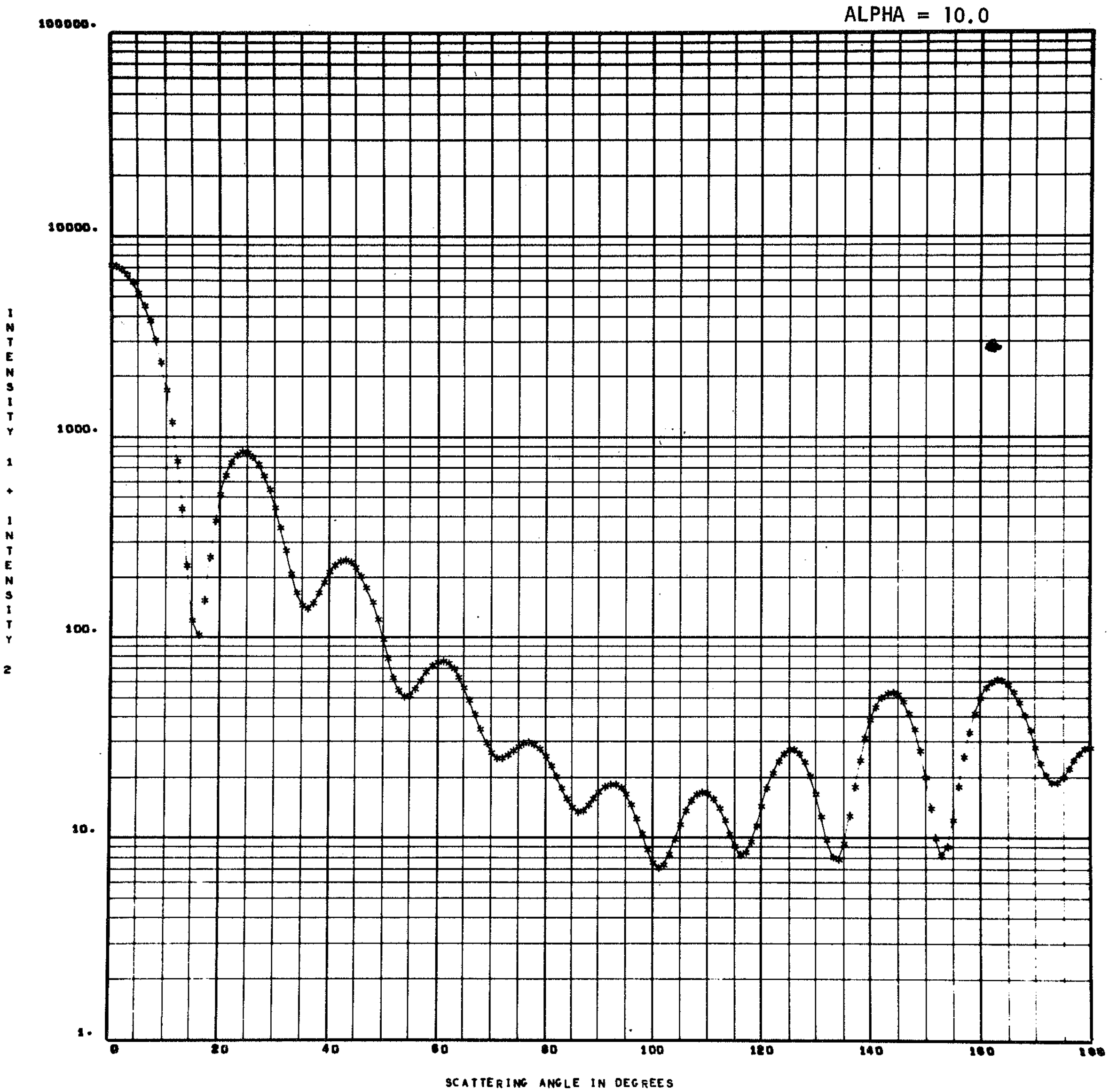


ALPHA = 10.0
1000.00
100.00
10.00
1.00
0.10
0
20
40
60
80
100
120
140
160
180
INTENSITY 1 (* AND INTENSITY 2 +
THETA DEGREES


ALPHA = 10.0
1000.00
100.00
10.00
1.00
0.10
INTENSITY 1 * INTENSITY 2
0
20
40
60
80
100
120
140
THETA DEGREES


ALPHA = 10.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
POLARIZATION
0
20
40
60
80
100
120
140
160
180
200
THETA DEGREES


ALPHA = 10.0
100000.
10000.
1000.
100.
10.
1.
INTENSITY 1 * AND INTENSITY 2 +
0
20
40
60
80
100
120
140
160
180
SCATTERING ANGLE IN DEGREES



ALPHA = 10.0
100000.
10000.
1000.
100.
10.
1.
INTENSITY 1 + INTENSITY 2
0
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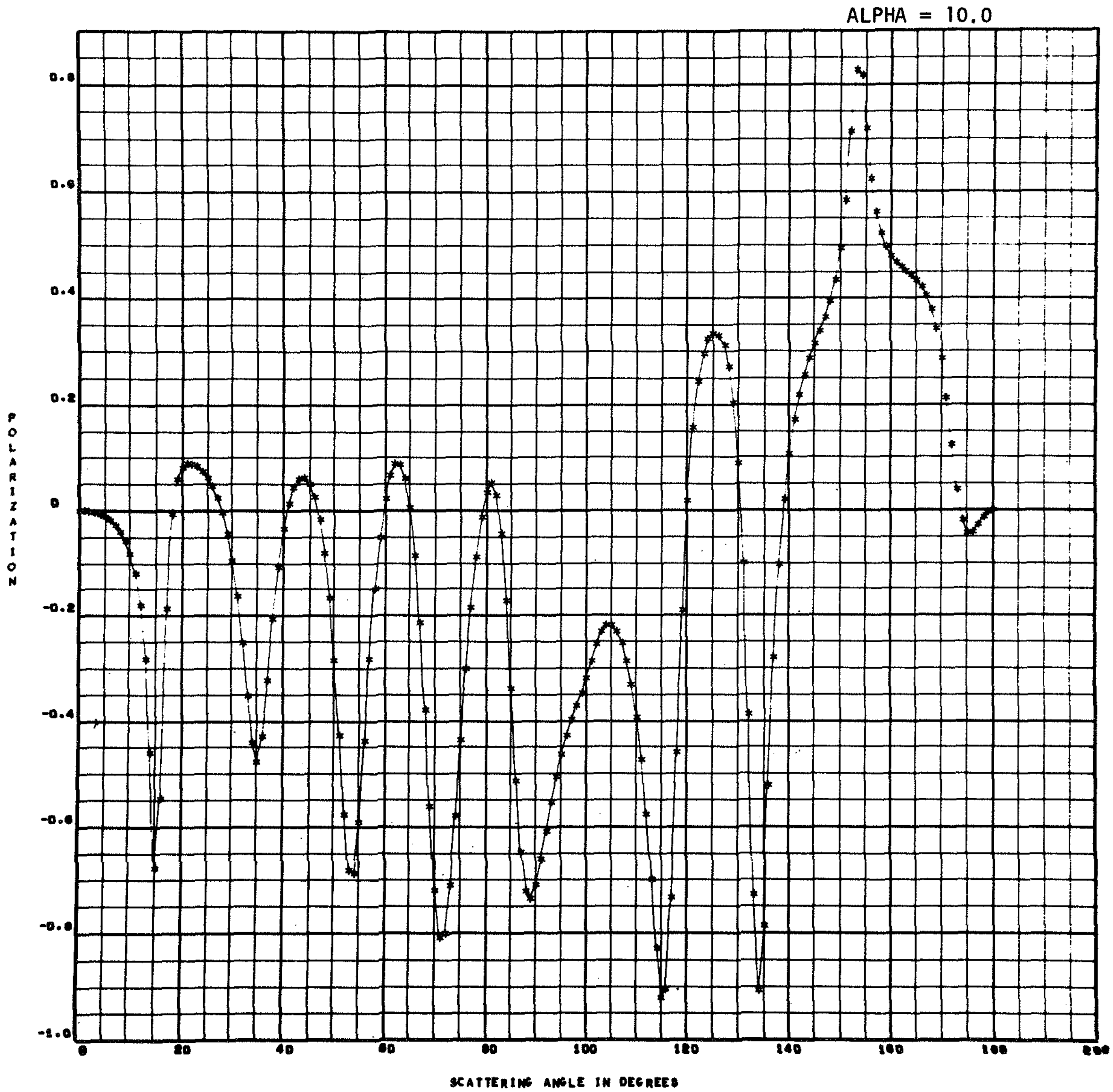
100
120
140
160
180
SCATTERING ANGLE IN DEGREES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0
20
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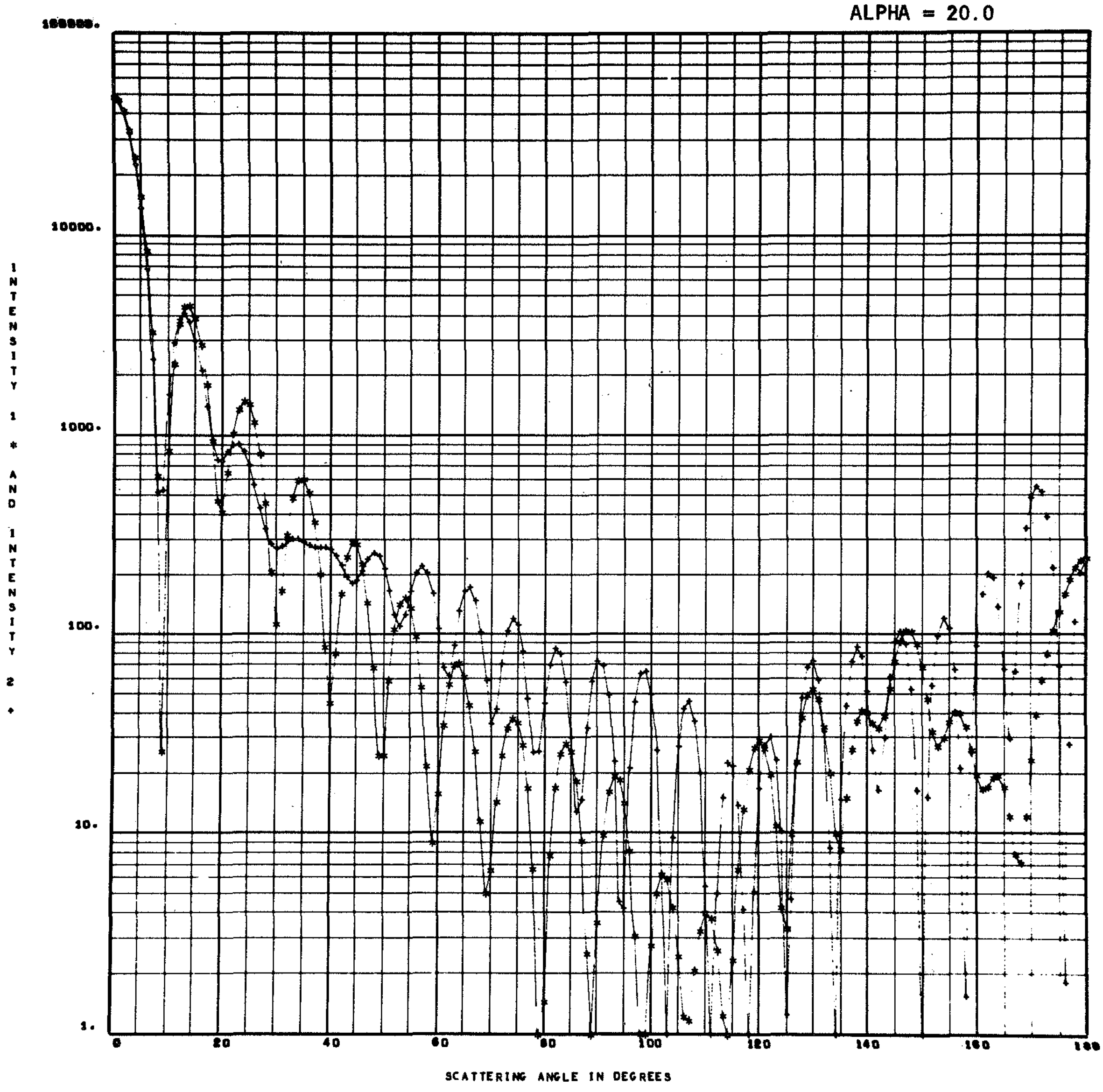
140
160
180
200
SCATTERING ANGLE IN DEGREES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10.
1.
0
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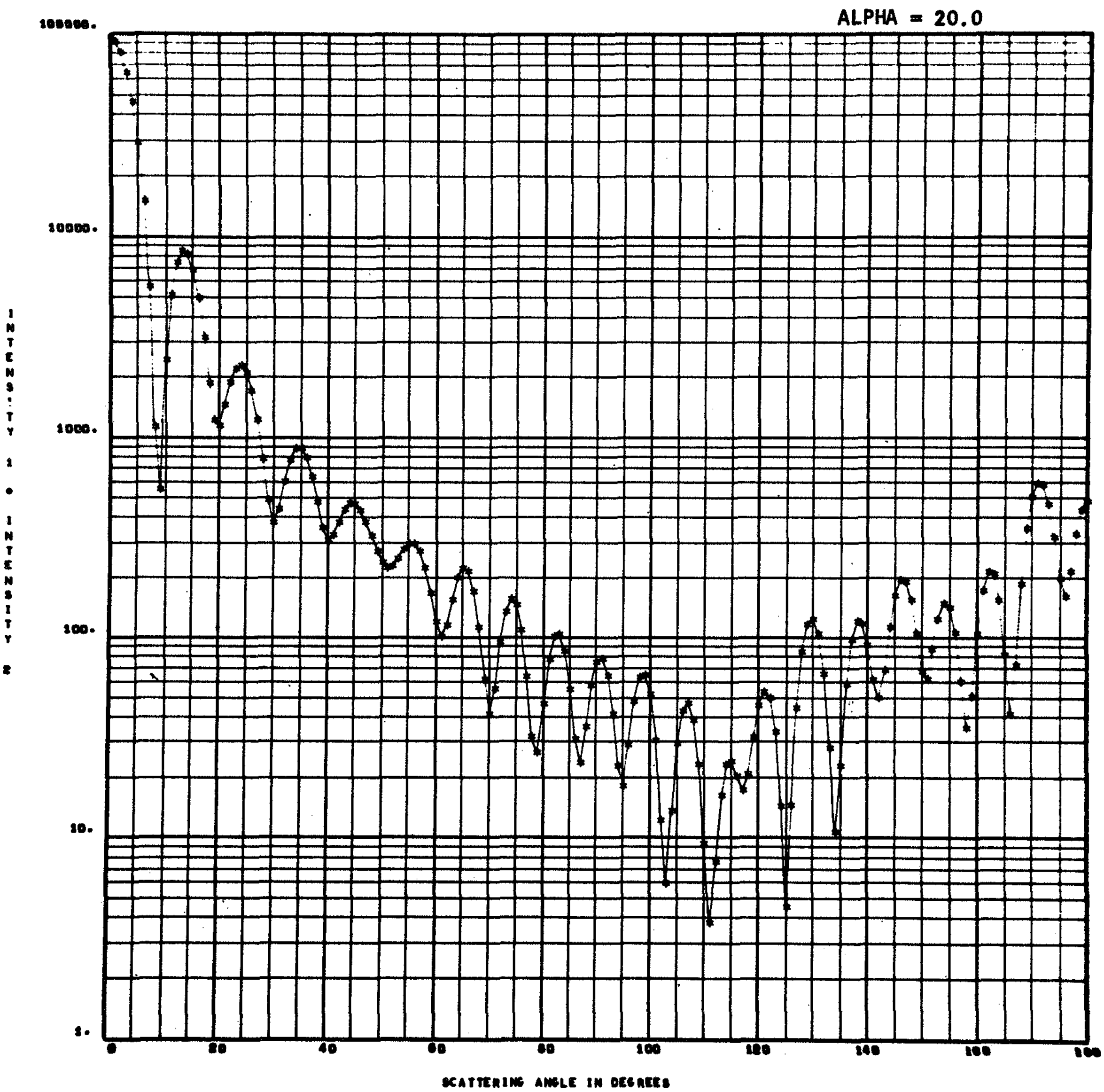
SCATTERING ANGLE IN DEGREES
INTENSITY 1 * AND INTENSITY 2 +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INTENSITY 1 + INTENSITY 2
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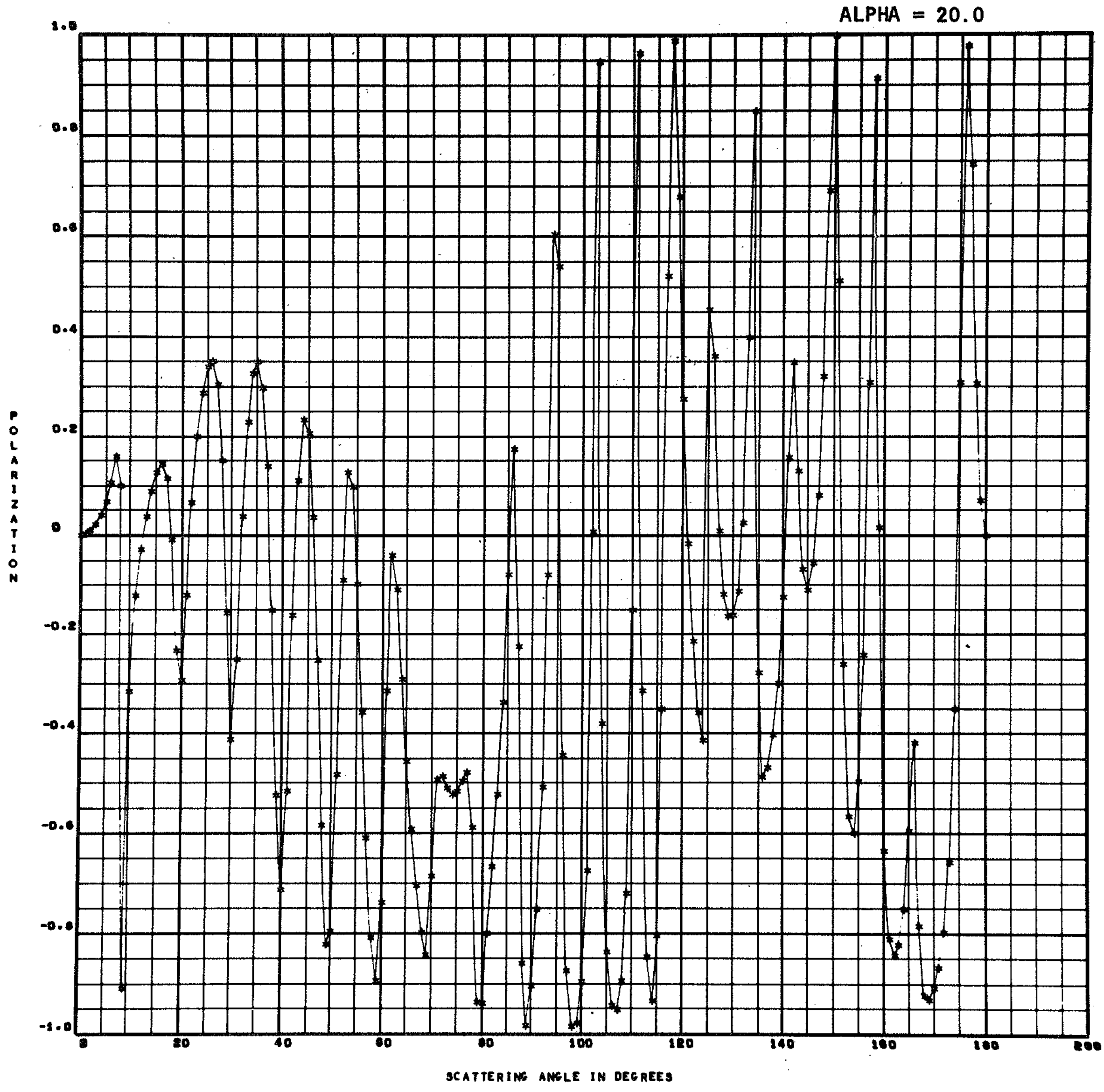
120
140
160
180
SCATTERING ANGLE IN DEGREES



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-0.6
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-1.0
POLARIZATION
0
20
40
60
80
100
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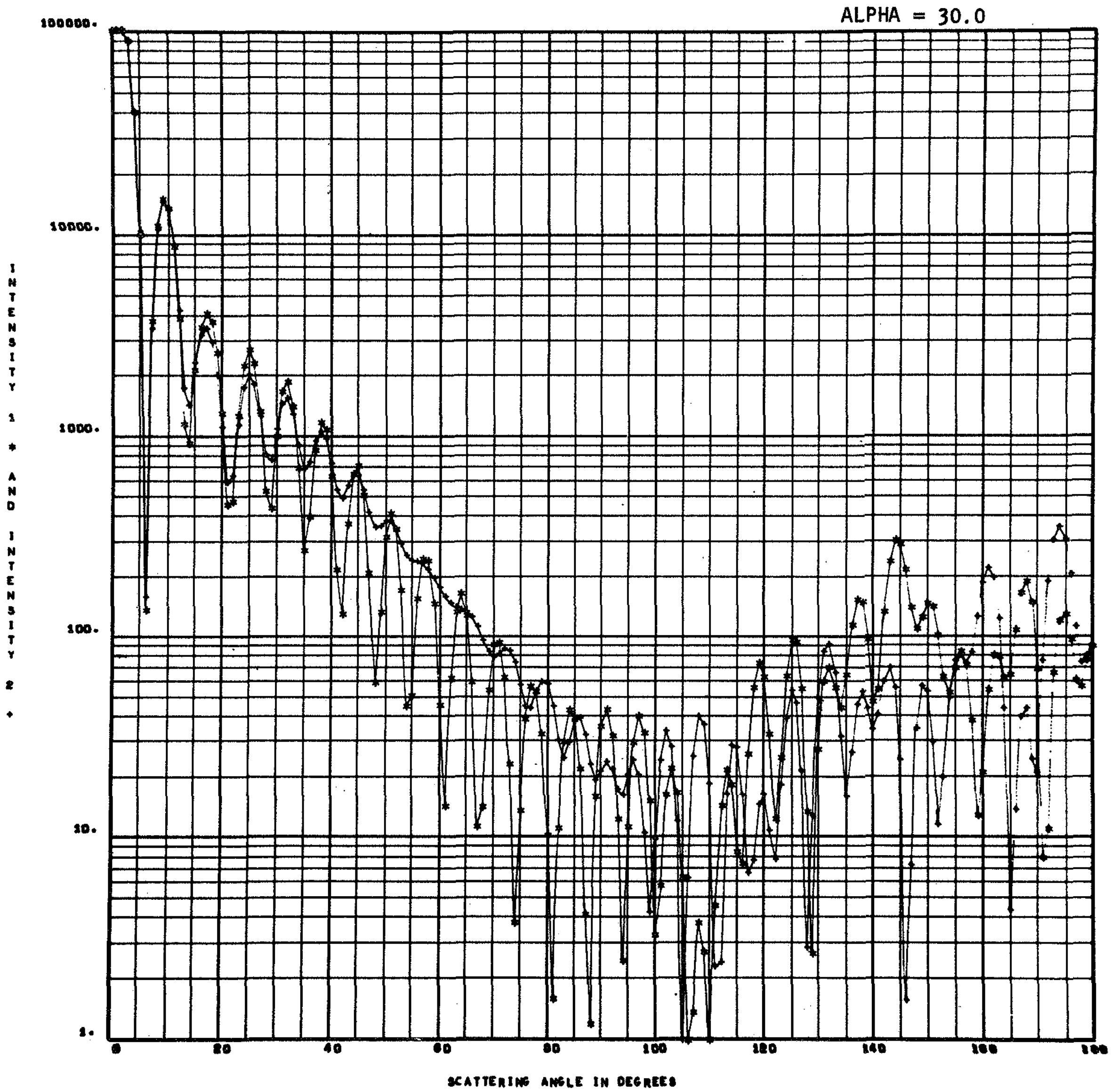
140
160
180
200
SCATTERING ANGLE IN DEGREES



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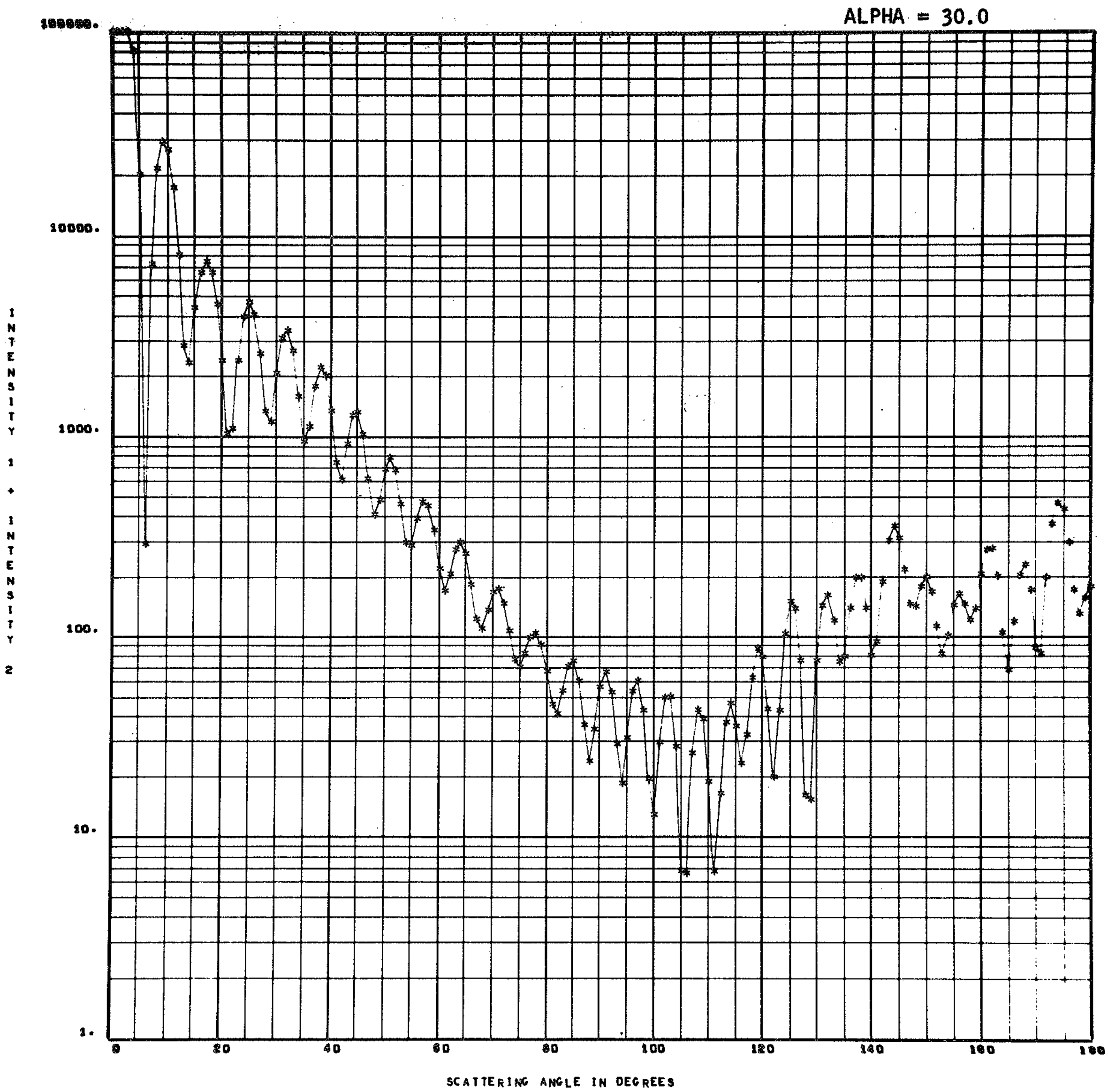
160
180
SCATTERING ANGLE IN DEGREES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1.
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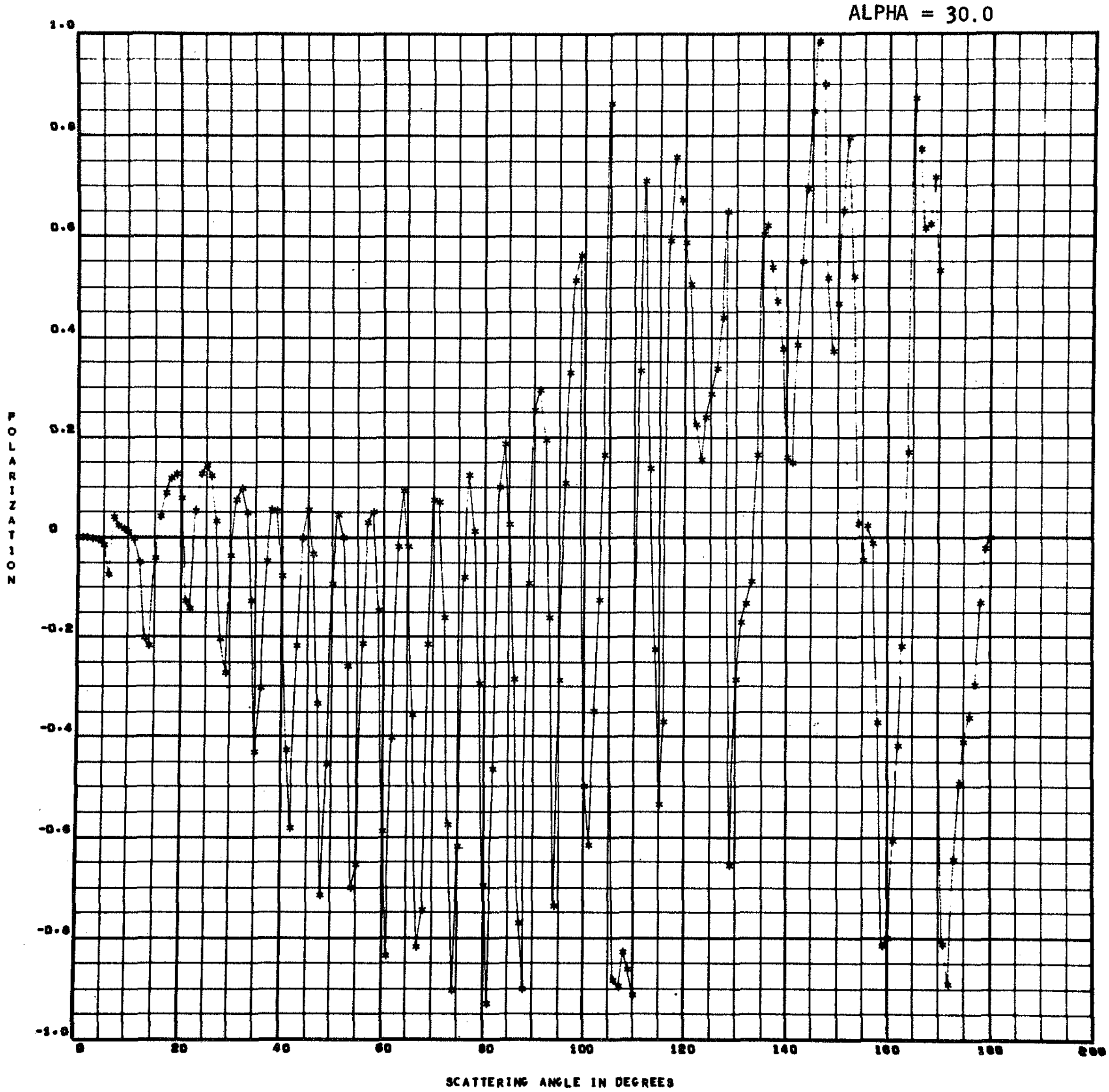
120
140
160
180
SCATTERING ANGLE IN DEGREES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SCATTERING ANGLE IN DEGREES
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0.2
0
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-0.4
-0.6
-0.8
-1.0
0
20
40
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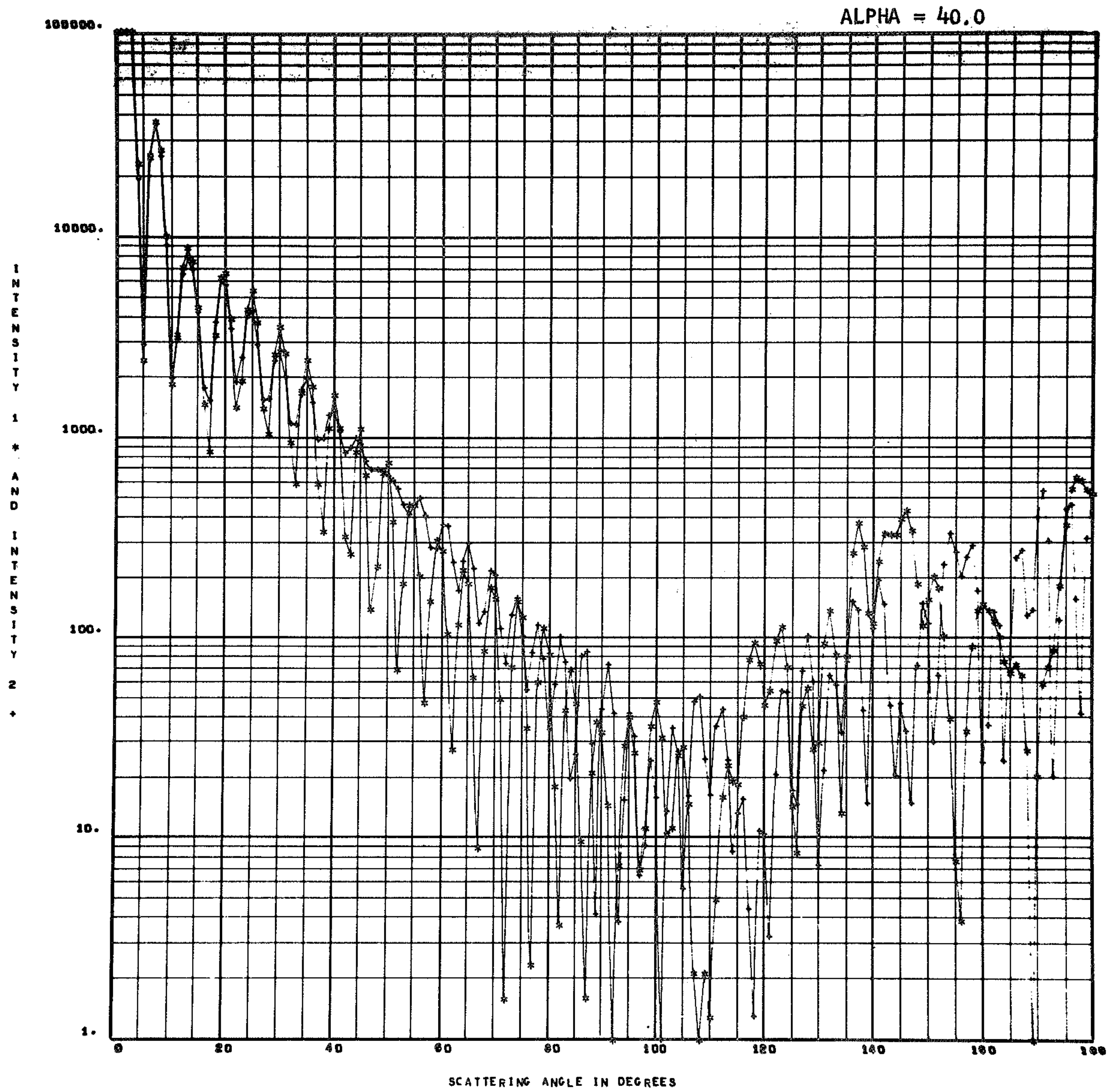
100
120
140
160
180
200



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INTENSITY 1 * AND INTENSITY 2 +
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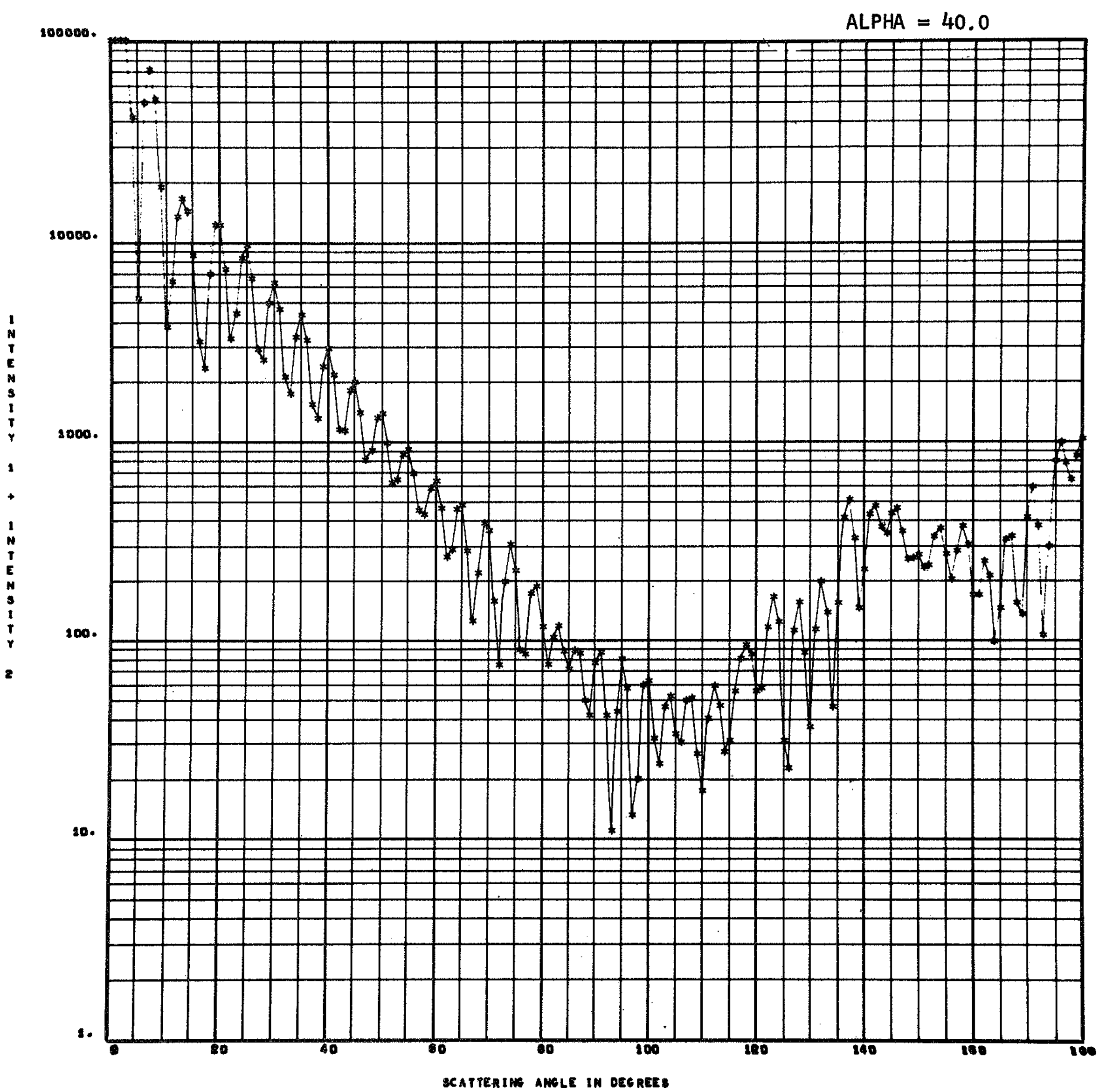
160
180
SCATTERING ANGLE IN DEGREES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1.
INTENSITY 1 + INTENSITY 2
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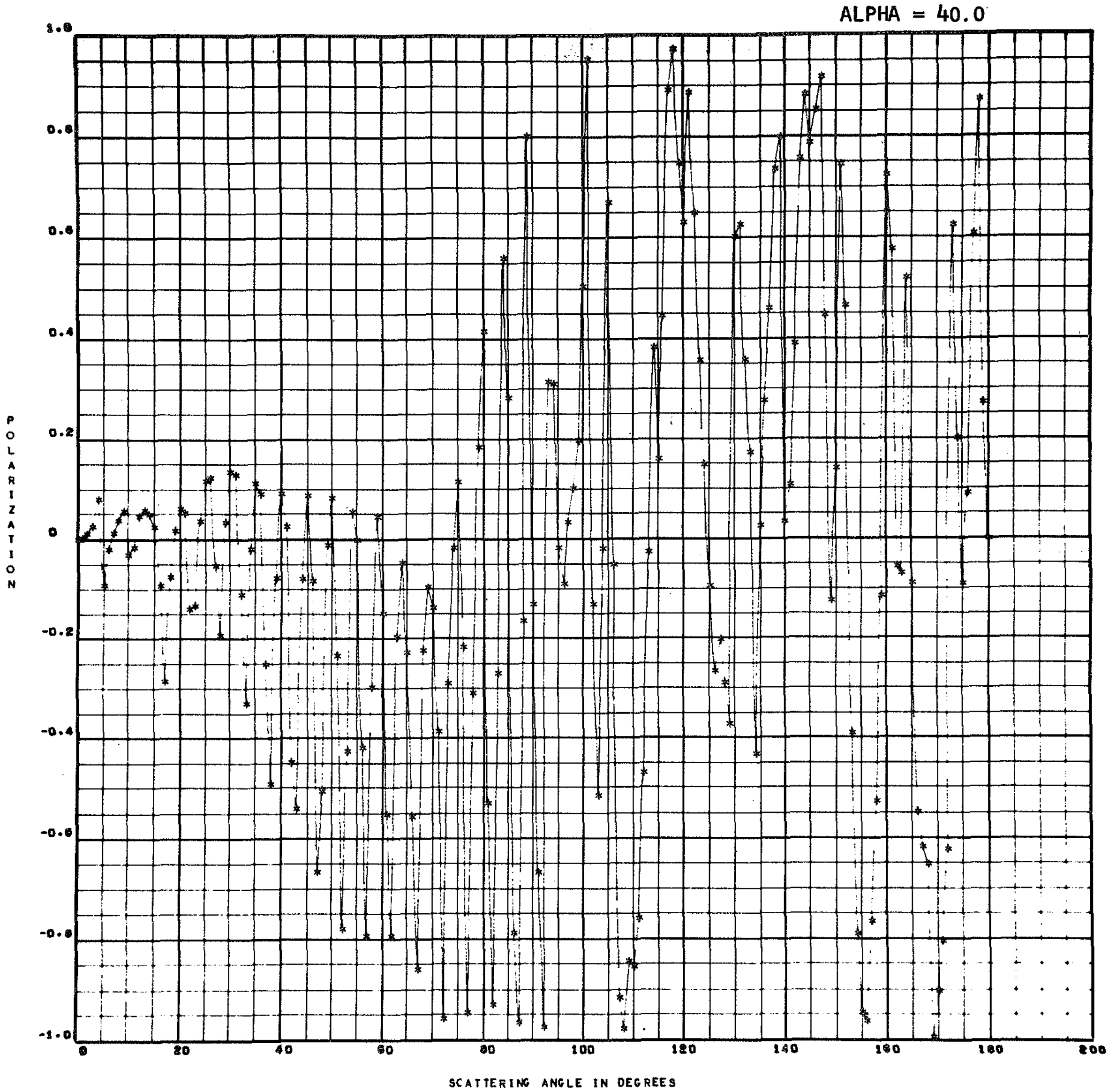
160
180
SCATTERING ANGLE IN DEGREES

ALPHA = 40.0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0
-0.2
-0.4
-0.6
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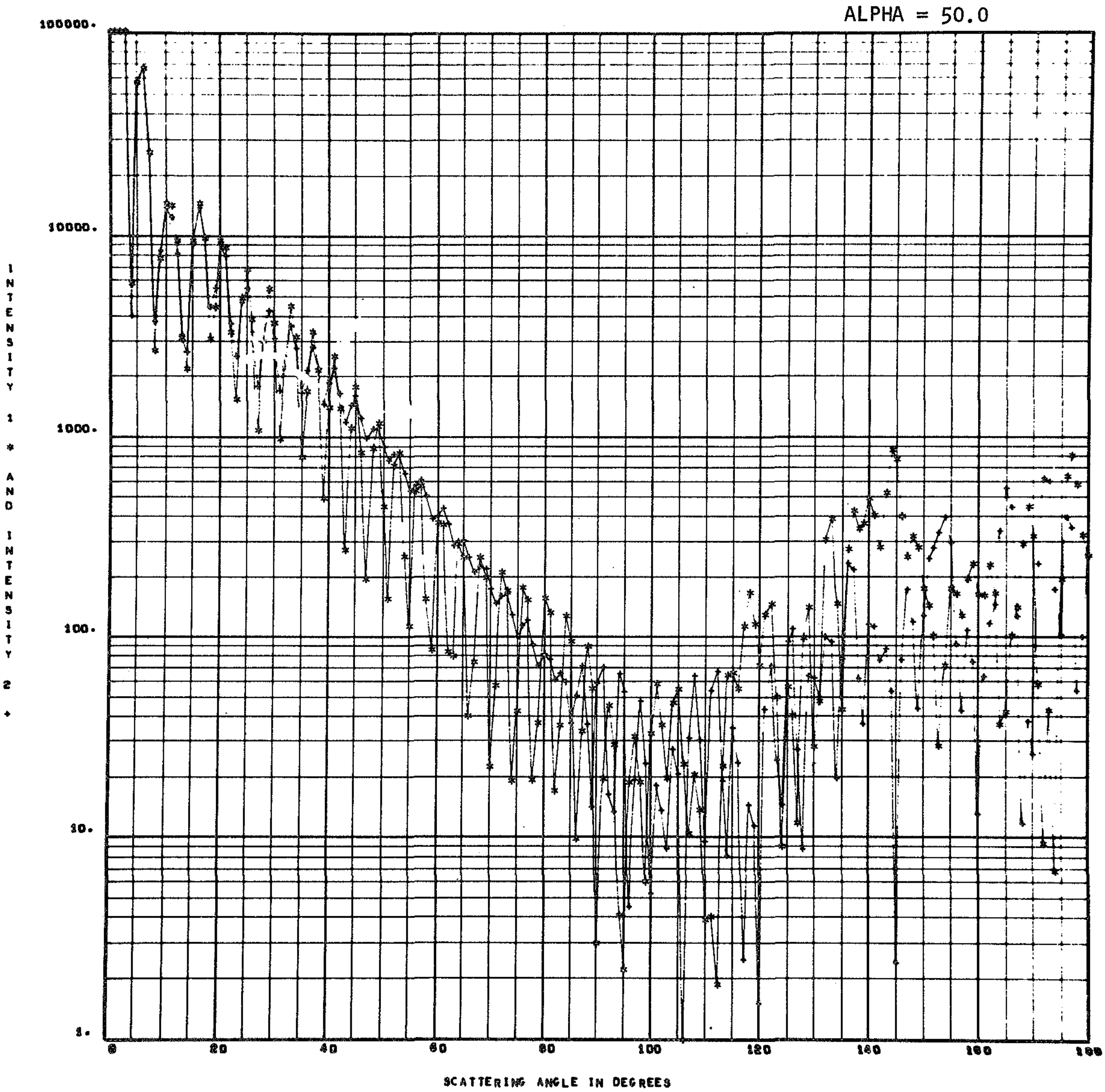
POLARIZATION
0
20
40
60
80
100
120
140
160
180
200
SCATTERING ANGLE IN DEGREES



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100
120
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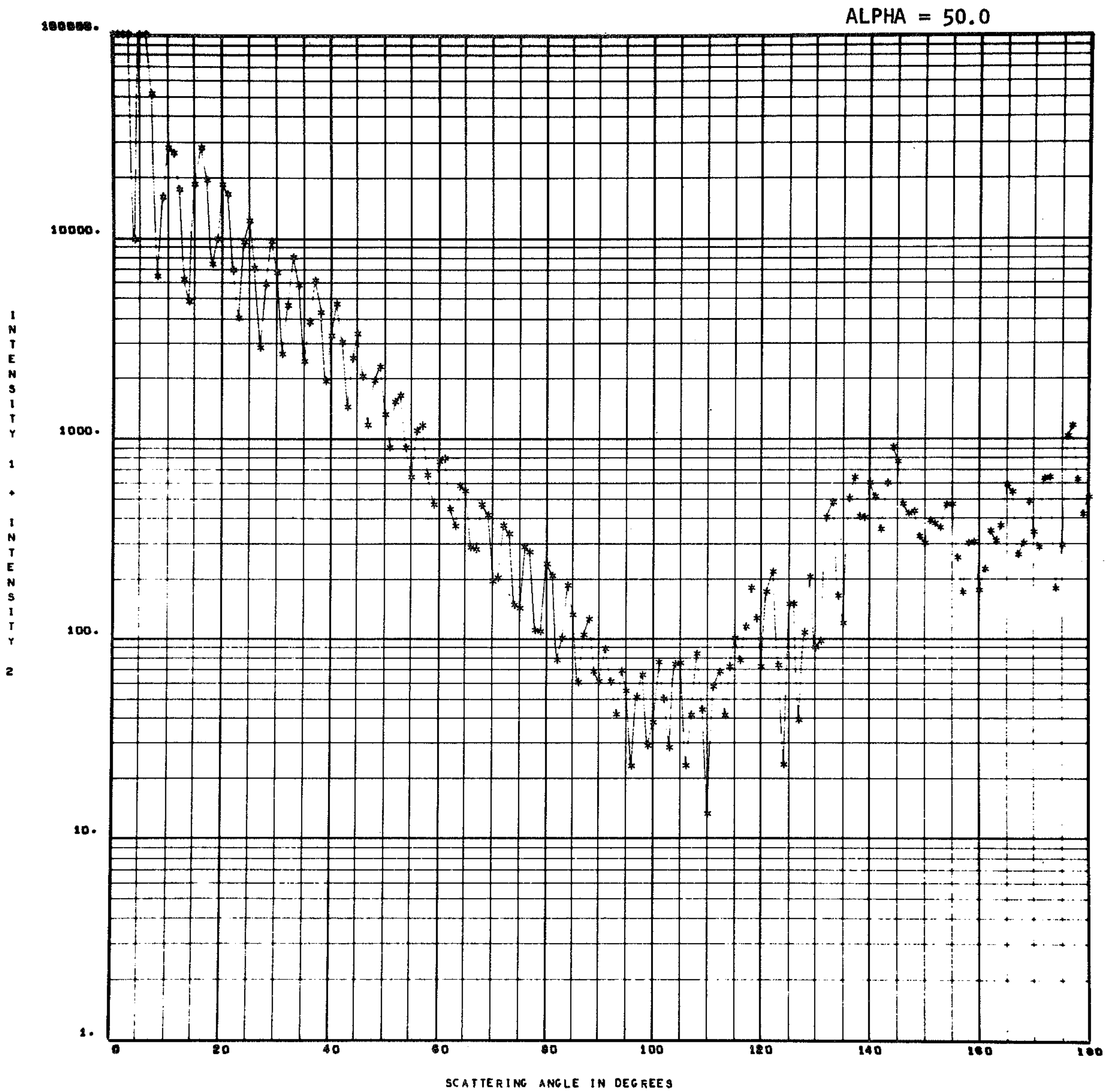
160
180
SCATTERING ANGLE IN DEGREES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INTENSITY 1 + INTENSITY 2
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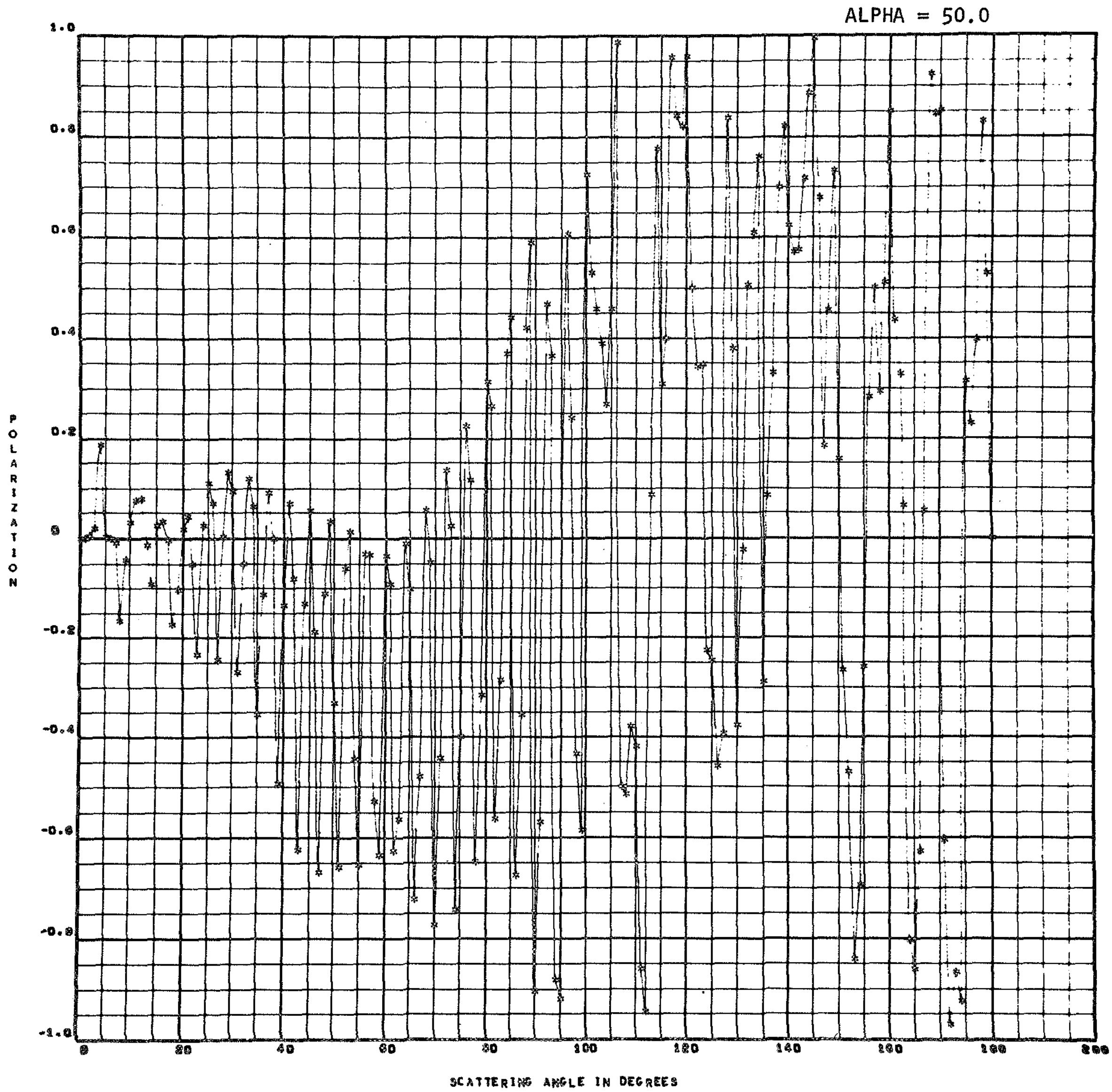
160
180
SCATTERING ANGLE IN DEGREES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60
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100
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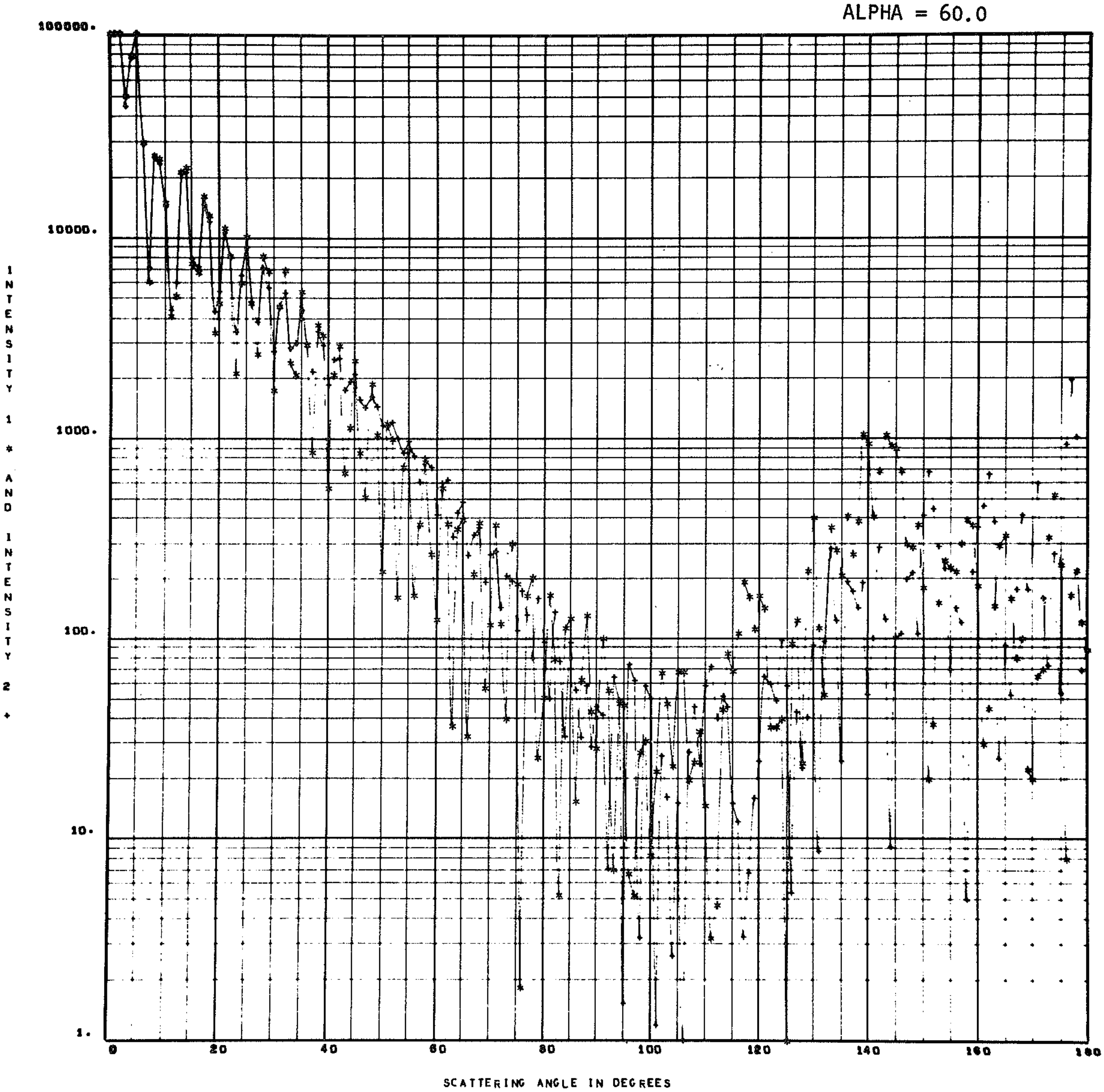
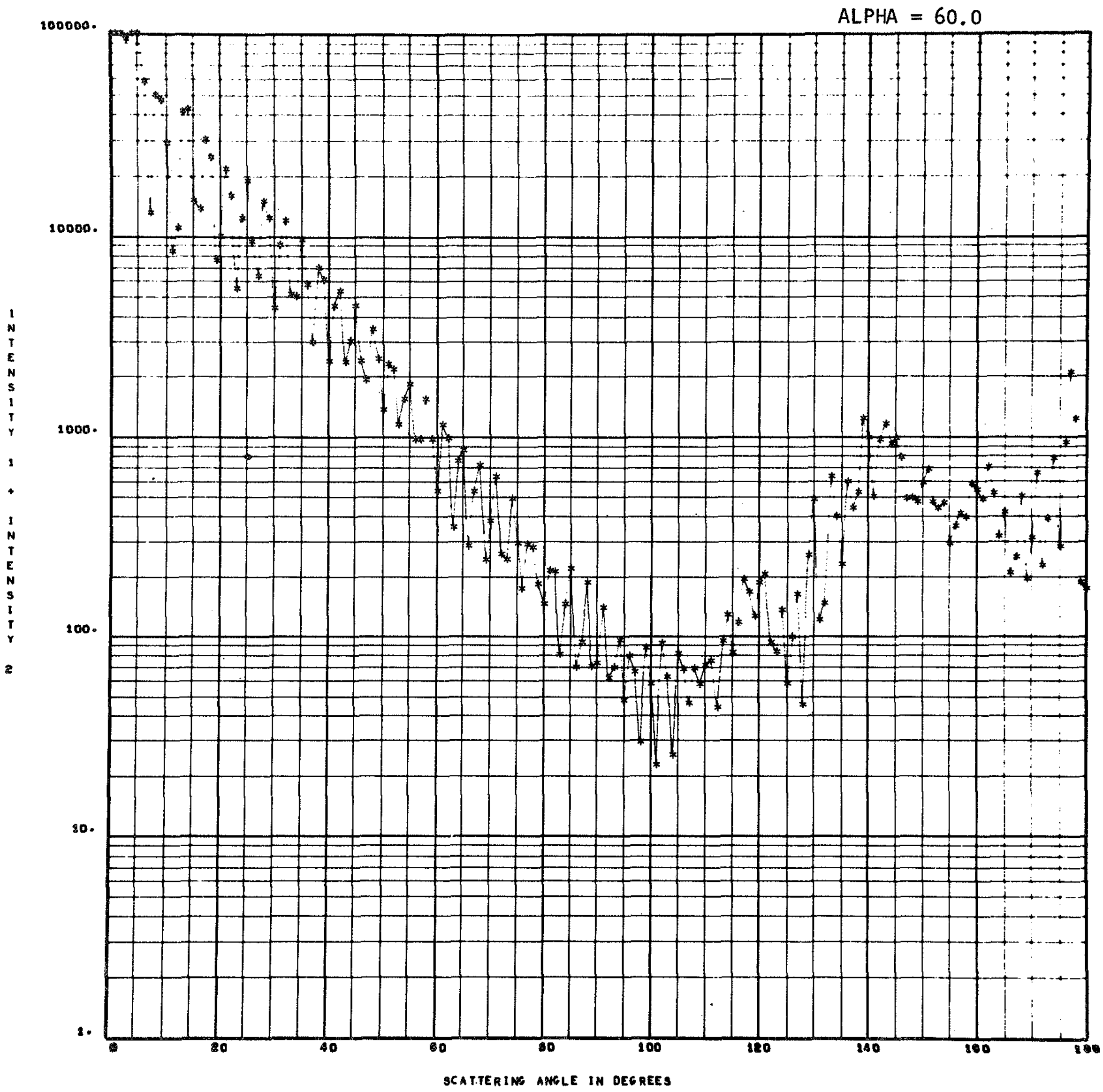
140
160
180
200
SCATTERING ANGLE IN DEGREES

ALPHA = 60.0



100000.
10000.
1000.
100.
10.
1.
0
20
40
60
80
100
120
140
160
180
INTENSITY 1 * AND INTENSITY 2 +
SCATTERING ANGLE IN DEGREES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1.
INTENSITY 1 + INTENSITY 2
0
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40
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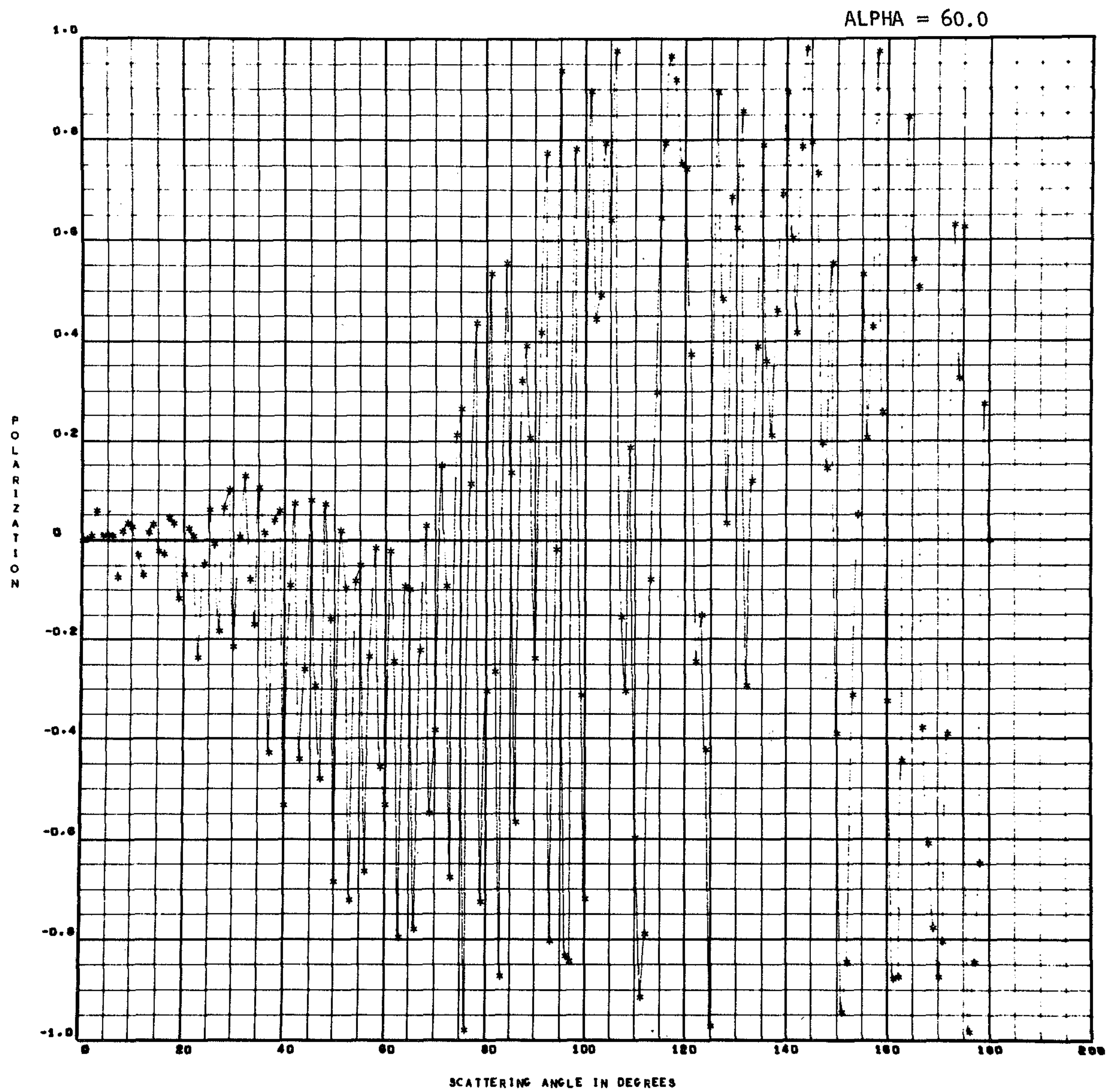
160
180
SCATTERING ANGLE IN DEGREES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-0.6
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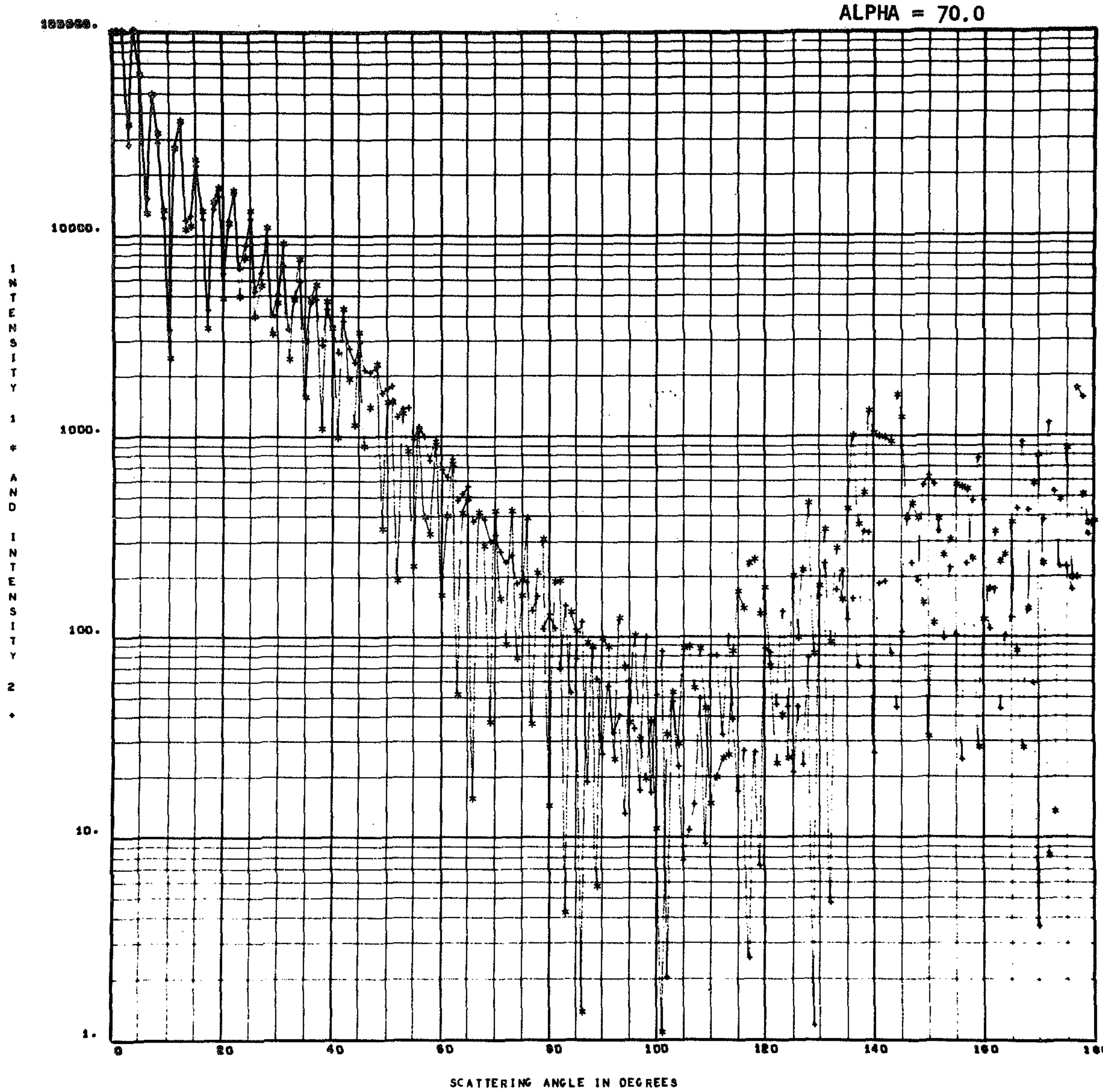
POLARIZATION
0
20
40
60
80
100
120
140
160
180
200
SCATTERING ANGLE IN DEGREES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1.
INTENSITY 1 * AND INTENSITY 2 +
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60
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100
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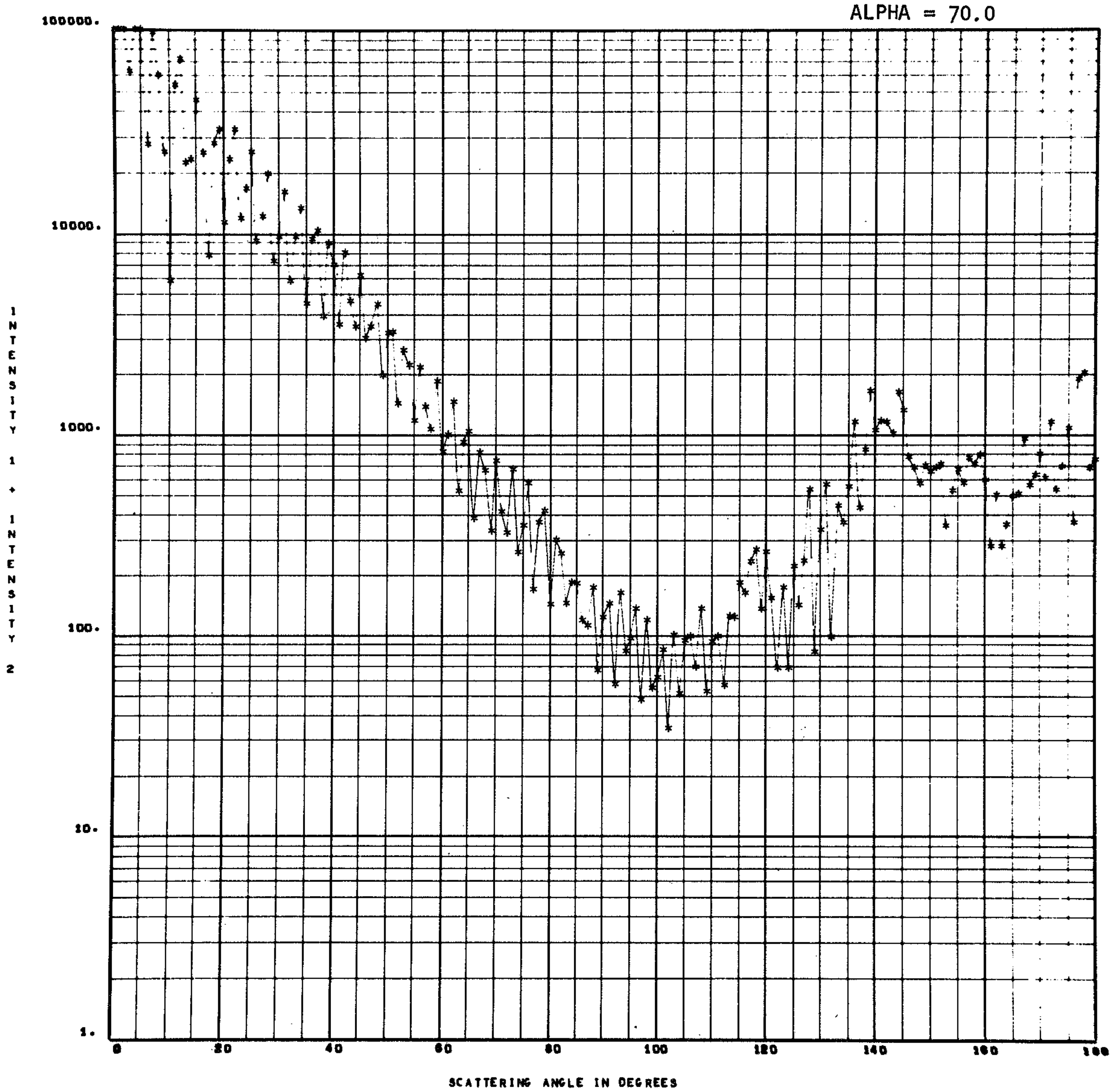
160
180
SCATTERING ANGLE IN DEGREES

ALPHA = 70.0



100000.
10000.
1000.
100.
10.
1.
I N T E N S I T Y 1 + I N T E N S I T Y 2
0
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40
60
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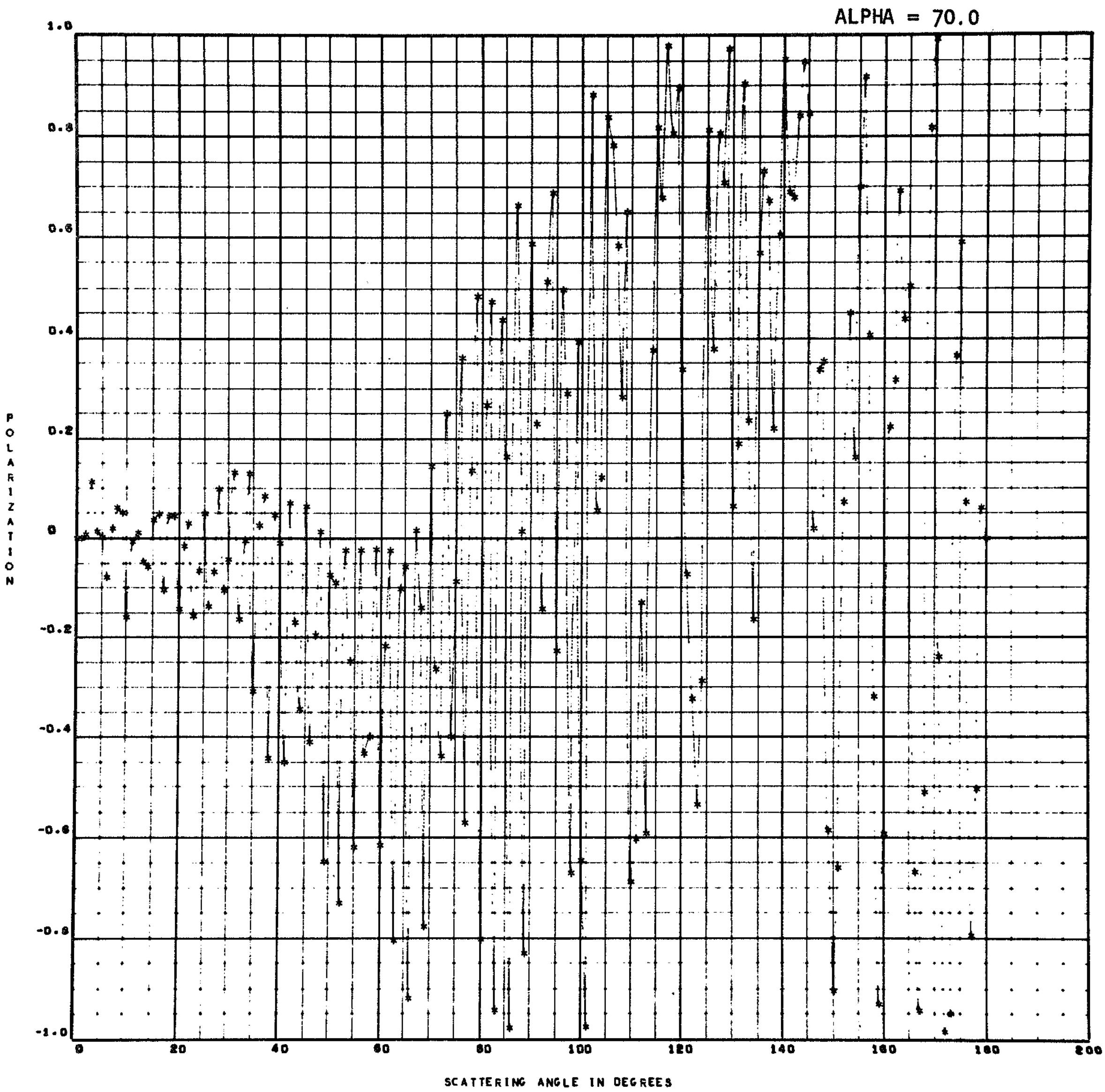
100
120
140
160
180
SCATTERING ANGLE IN DEGREES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POLARIZATION
SCATTERING ANGLE IN DEGREES
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0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
0
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100
120
140
160
180
200

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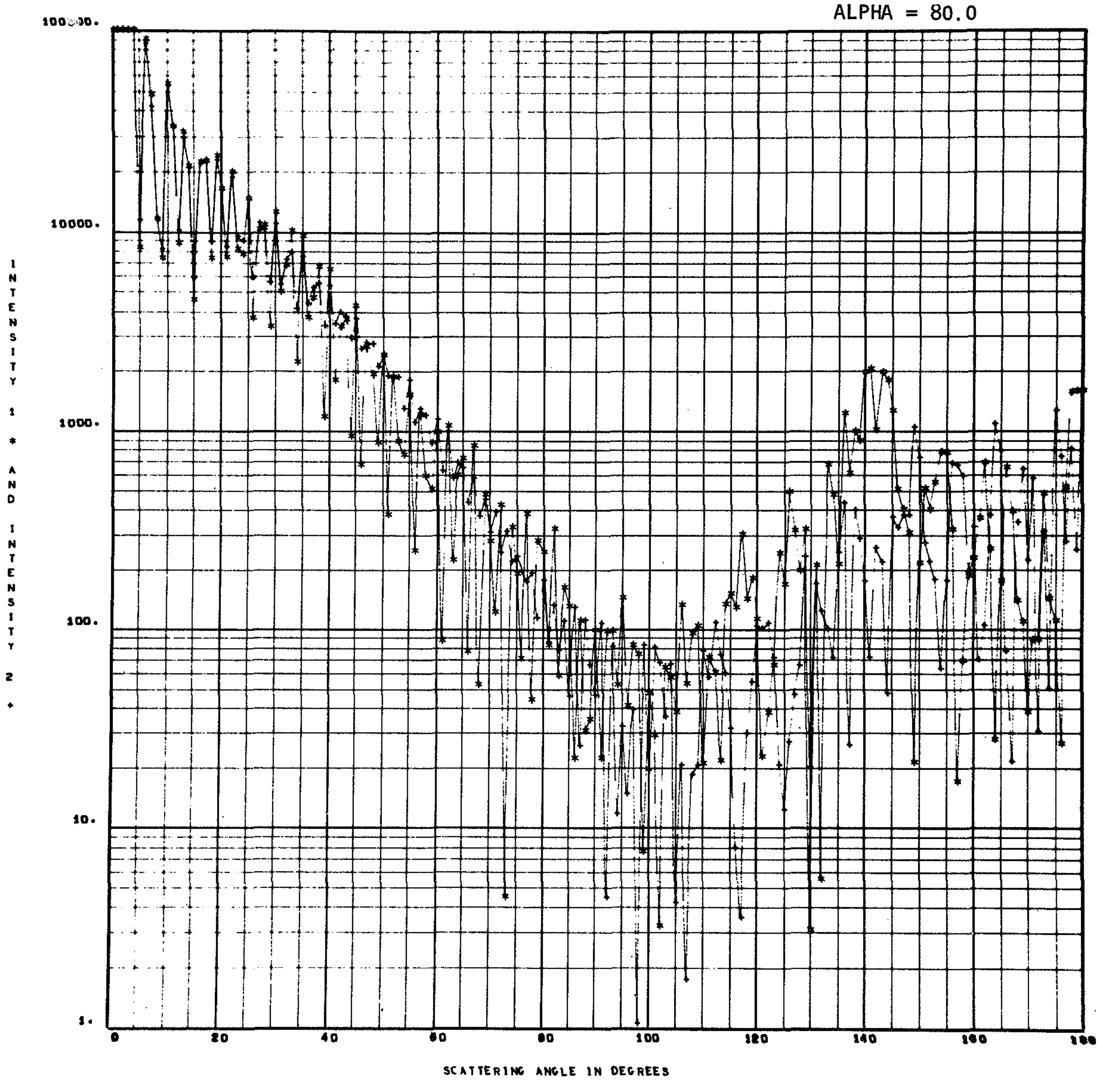

100000.
10000.
1000.
100.
10.
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INTENSITY 1 * AND INTENSITY 2 +
0
20
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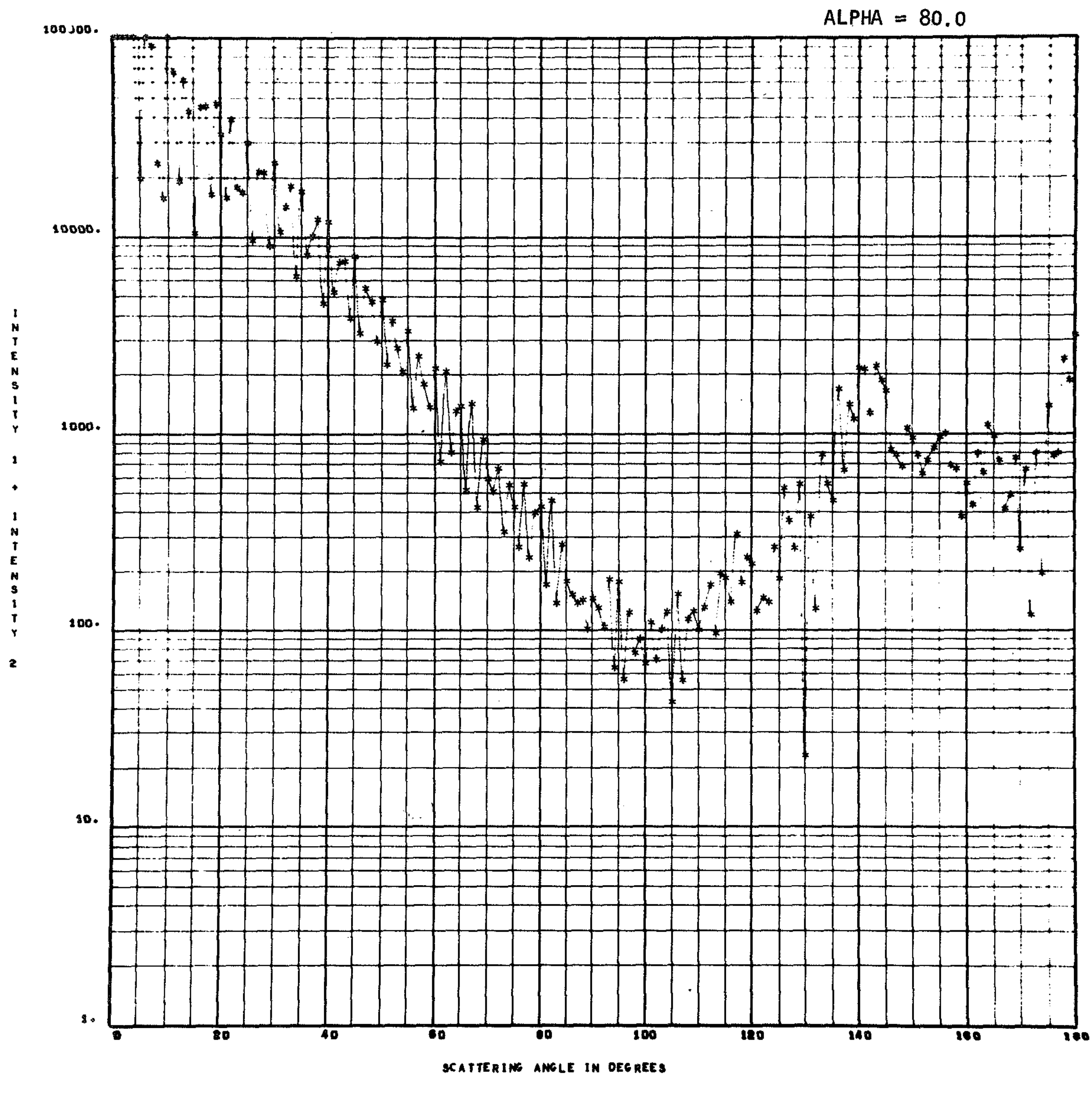
180
SCATTERING ANGLE IN DEGREES



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1000.
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INTENSITY 1 + INTENSITY 2
0
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120
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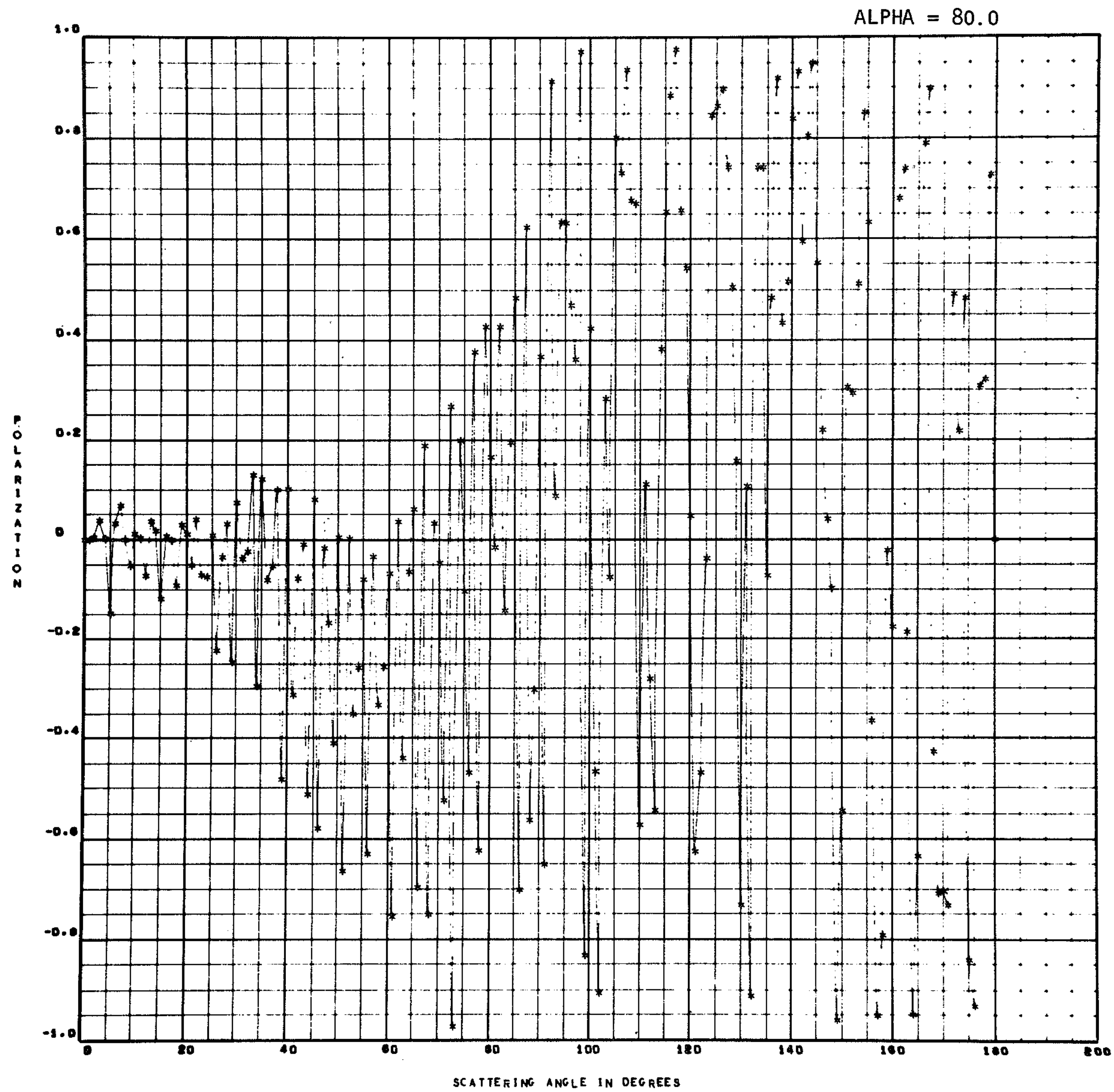
160
180
SCATTERING ANGLE IN DEGREES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POLARIZATION
0
20
40
60
80
100
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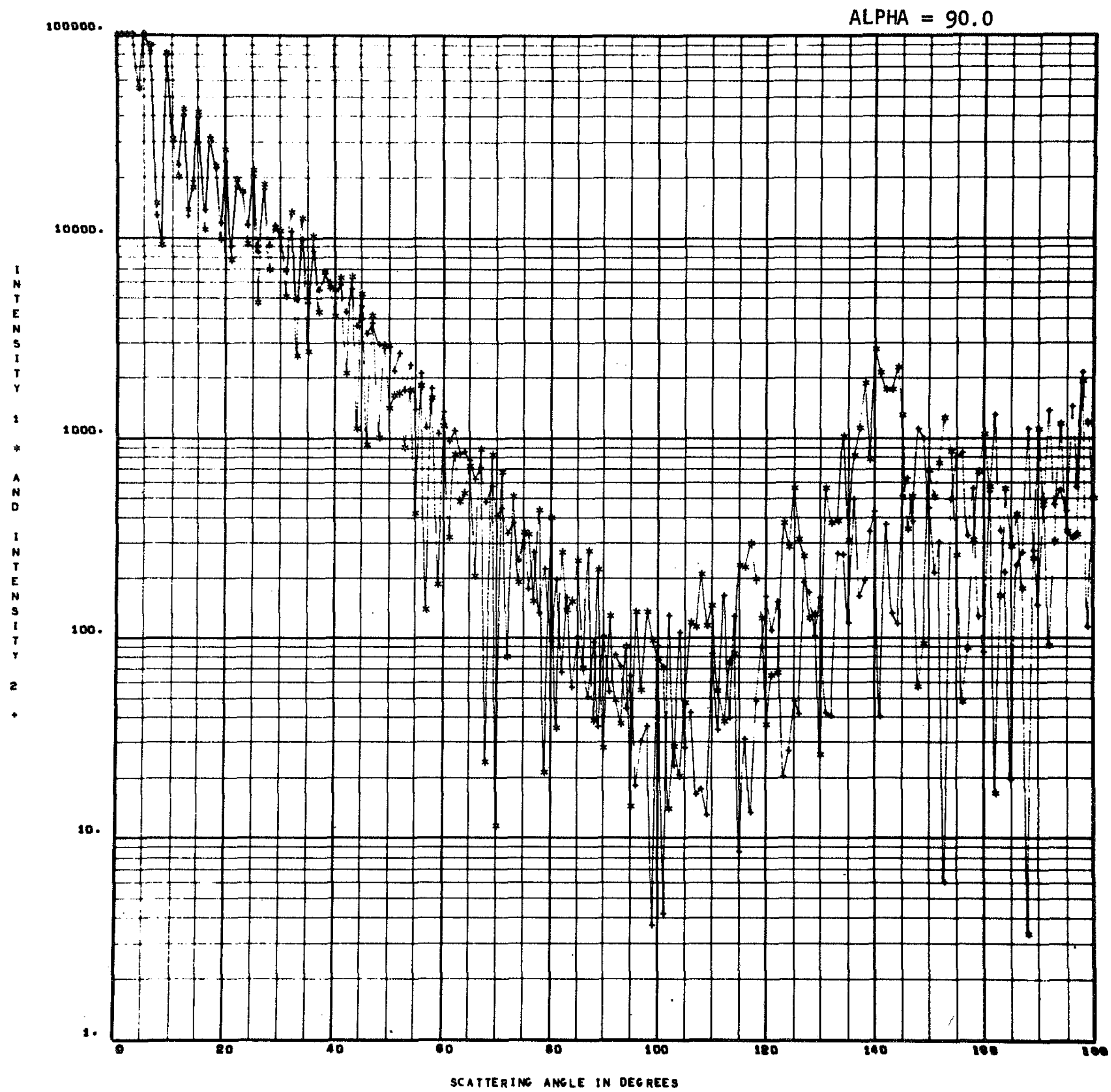
140
160
180
200
SCATTERING ANGLE IN DEGREES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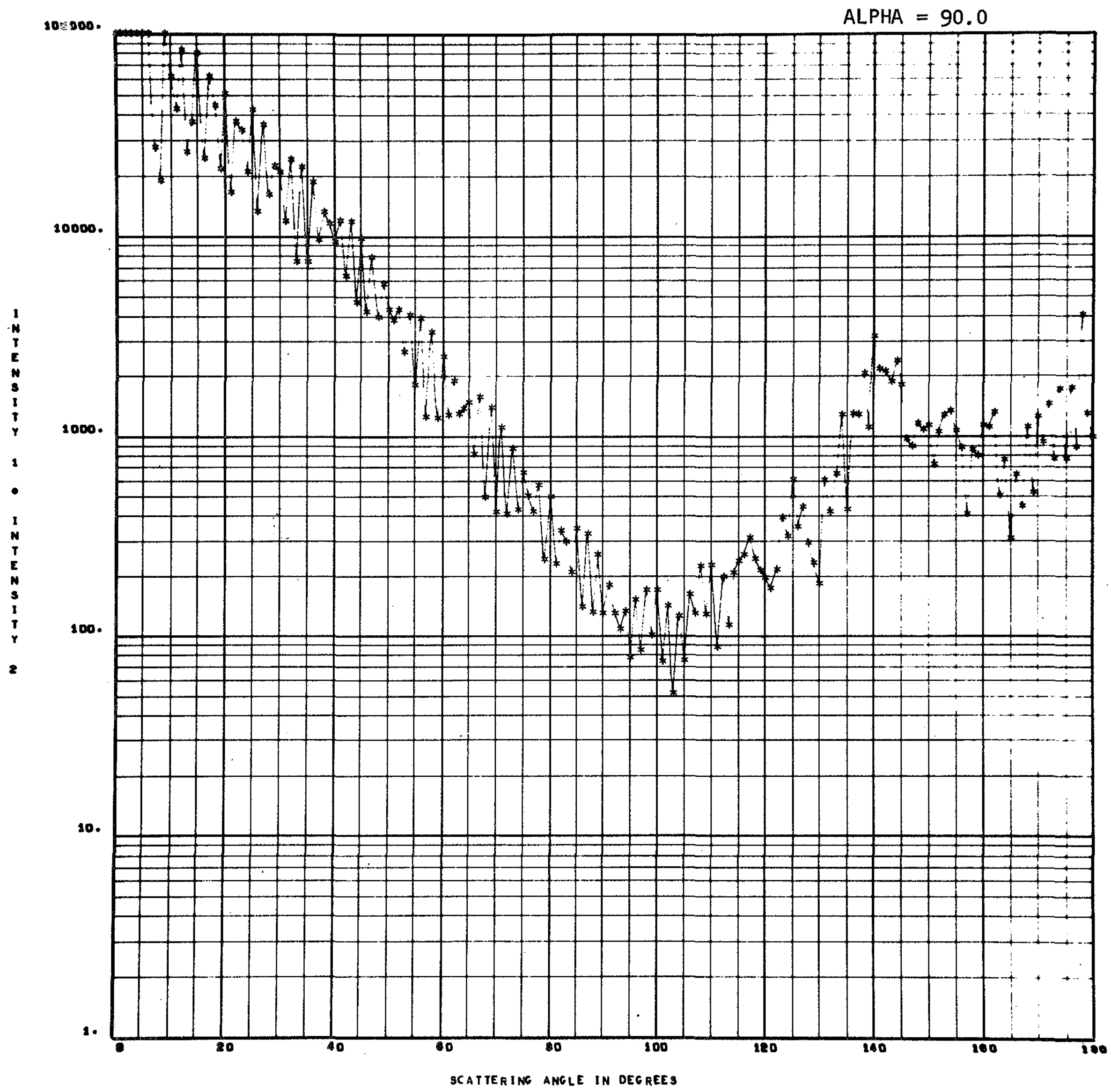
100
120
140
SCATTERING ANGLE IN DEGREES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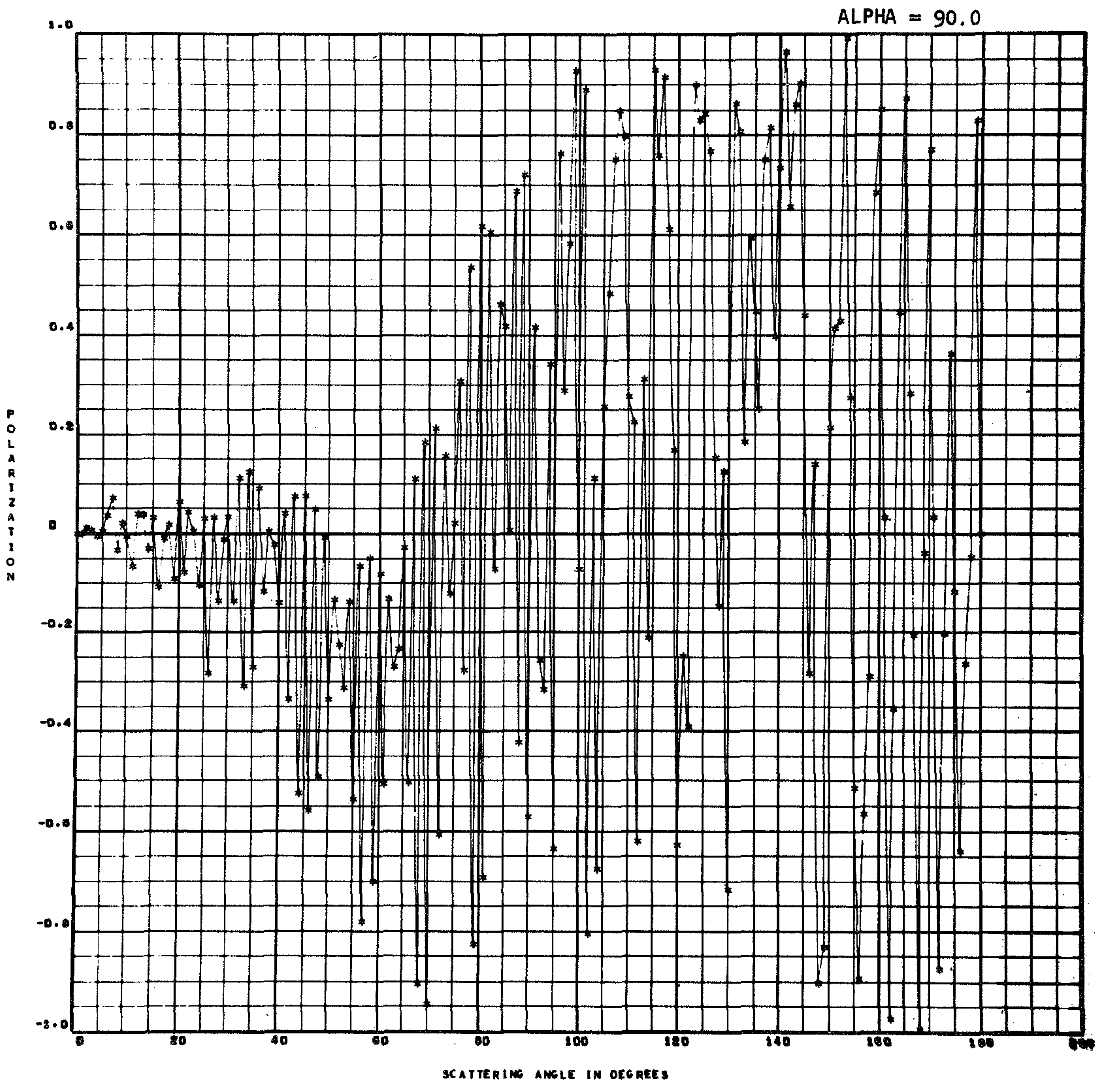
SCATTERING ANGLE IN DEGREES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160
180
200
SCATTERING ANGLE IN DEGREES

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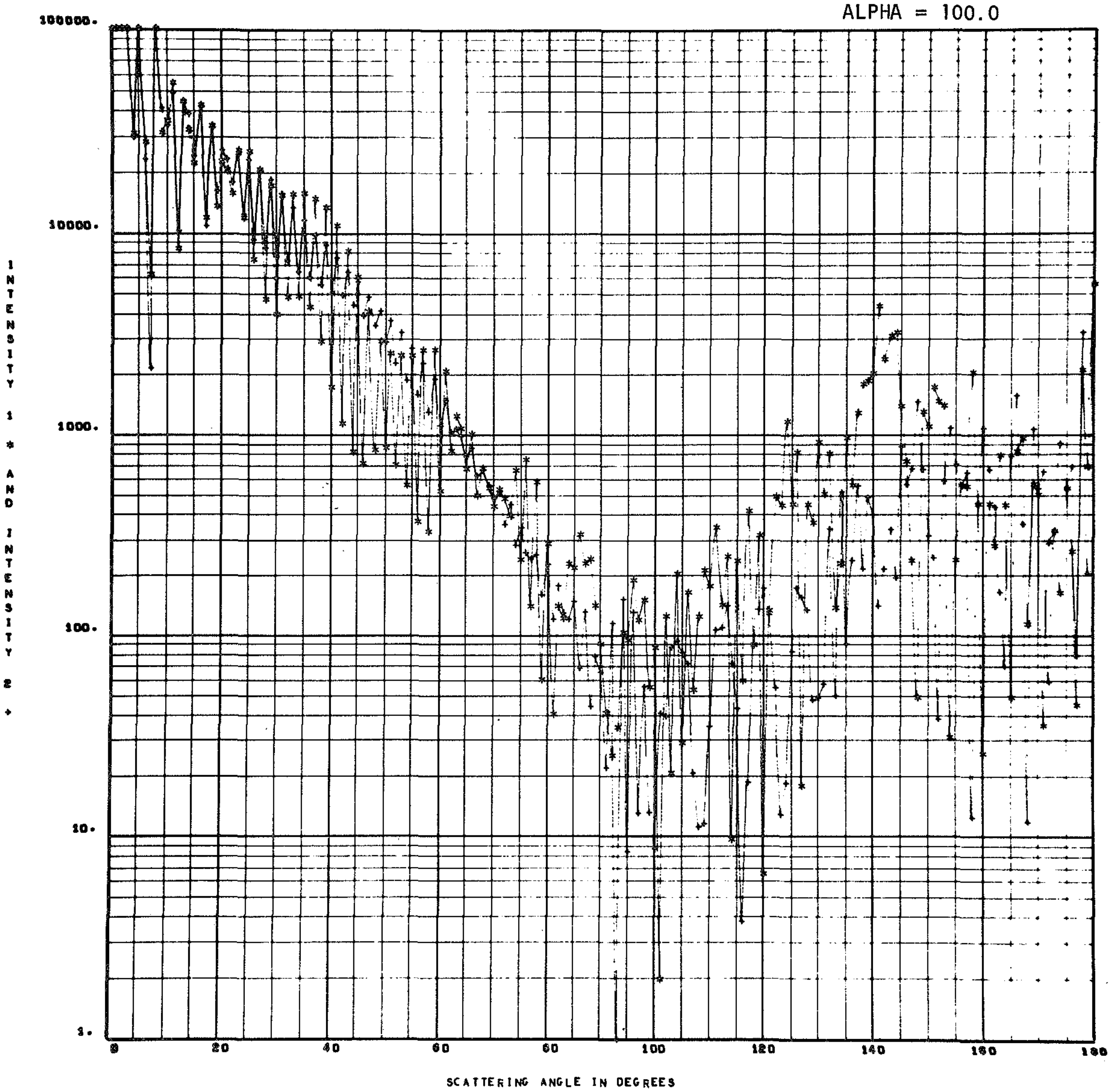

100000.
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INTENSITY 1 * AND INTENSITY 2 +
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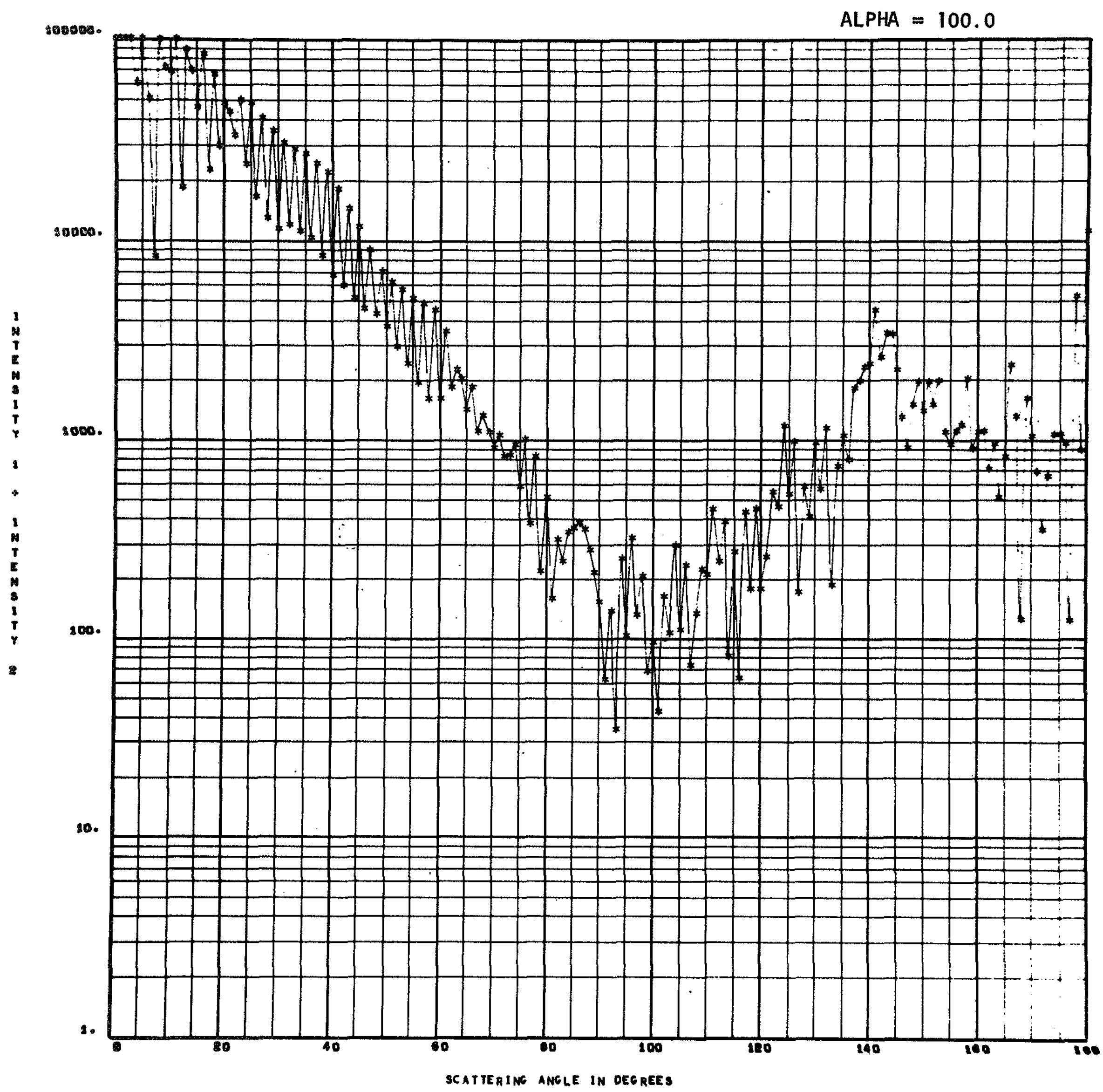
SCATTERING ANGLE IN DEGREES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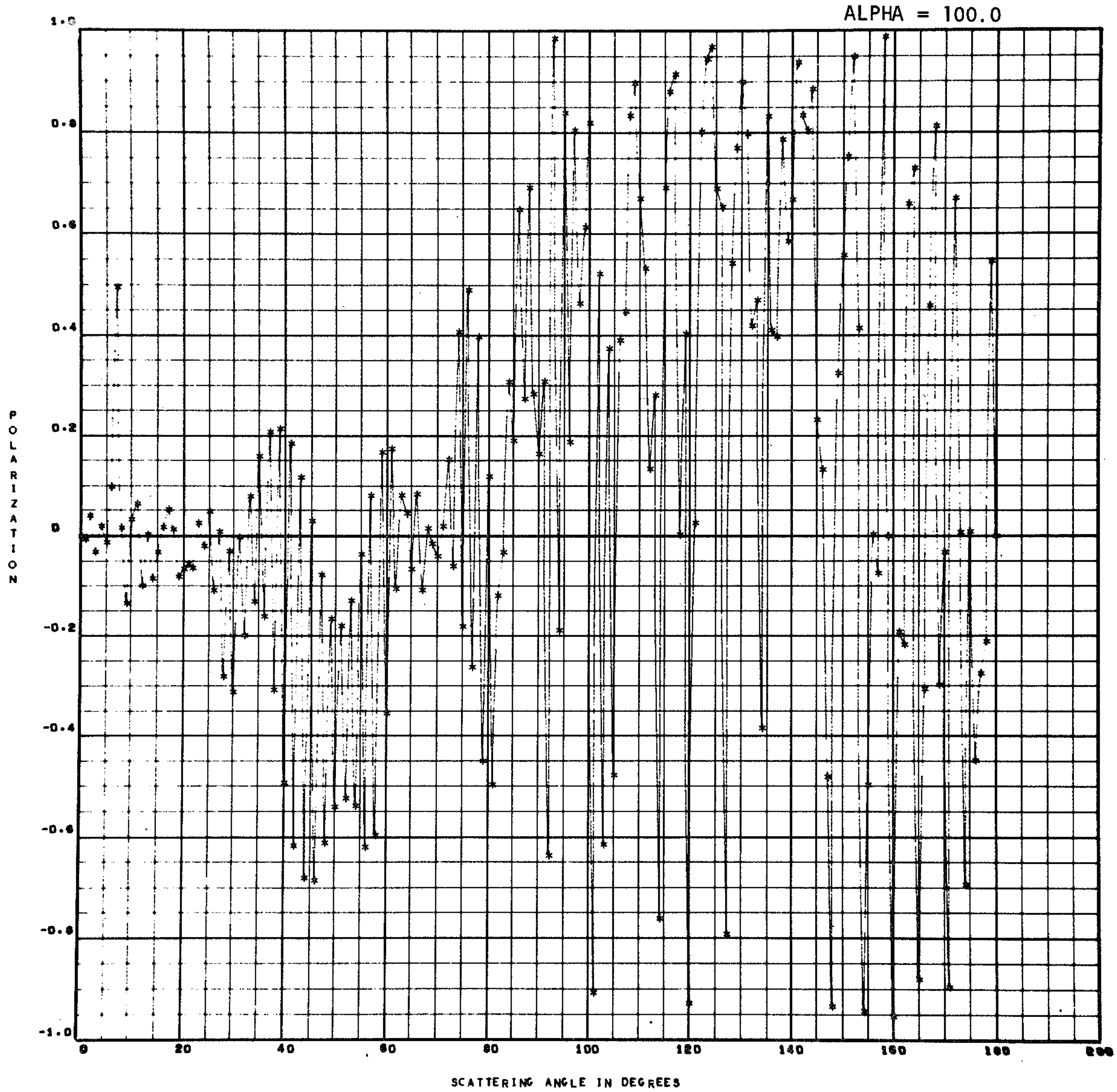
180
SCATTERING ANGLE IN DEGREES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-0.6
-0.8
-1.0
0
20
40
60
80
100
120
140
160
180
200
SCATTERING ANGLE IN DEGREES

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APPROVAL

TM X-53951

MIE SCATTERING: A COMPUTER PROGRAM AND AN ATLAS

By Nadine A. Bicket and Gilmer A. Gary

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This document has also been reviewed and approved for technical accuracy.

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Chief, Space Thermophysics Division

Gerhard B. Heller

GERHARD B. HELLER
Director, Space Sciences Laboratory

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